Spectrum for Y=0 brane in planar AdS/CFT

Ryo Suzuki (ITF, Utrecht University)

with Zoltán Bajnok (Hungarian Academy of Science)

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Based on JHEP 1208 (2012) 149

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Boundary

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AdS/CFT for open and closed strings

AdS/CFT Correspondence

IIB string on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N} = 4\ SU(N)$ super Yang-Mills

should make the same prediction in the large N limit

with the identification $$\frac{\sqrt{\lambda}}{2\pi} = \frac{R^2}{2\pi\alpha'} \sim \sqrt{N g_{\mathrm{str}}} \leftrightarrow \lambda = N g_{\mathrm{YM}}^2$$

AdS/CFT Correspondence

IIB string on $\mathrm{AdS}_5 \times \mathrm{S}^5$ **and** $\mathcal{N} = 4\ SU(N)$ **super Yang-Mills**

should make the same prediction in the large N **limit**

with the identification $\dfrac{\sqrt{\lambda}}{2\pi} = \dfrac{R^2}{2\pi\alpha'} \sim \sqrt{N g_{\mathrm{str}}} \leftrightarrow \lambda = N g_{\mathrm{YM}}^2$

Strong Weak Duality

Semiclassical string	SYM perturbation
$\lambda \gg 1$	$\lambda \ll 1$

- Difficulty if we want to study AdS/CFT
- Advantage if we want to apply AdS/CFT

AdS/CFT Correspondence

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Strong Weak Duality

Semiclassical string	SYM perturbation
$\lambda \gg 1$	$\lambda \ll 1$

Integrability + superconformal symmetry

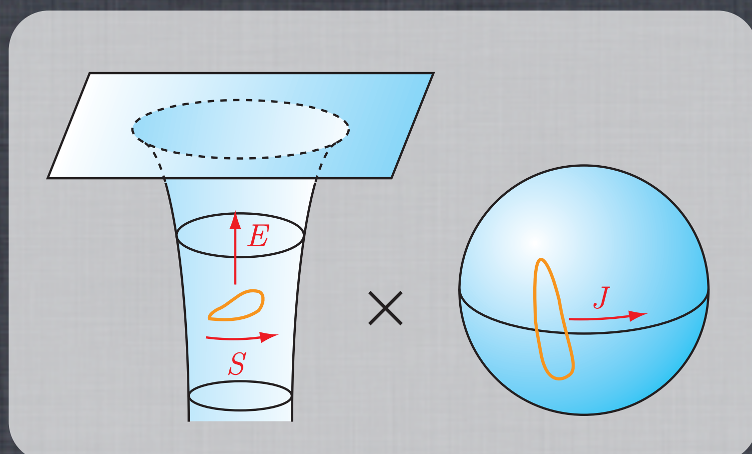
any λ

- Possible to test AdS/CFT by the exact computation

Most studied physical observables in AdS/CFT are

Closed string states $\leftrightarrow$ **Single-trace operators**

Energy of a short spinning string



$$E(\lambda)$$

Dimension of Konishi multiplet

$$\mathbf{tr}\left(\Phi^I\Phi^I\right)$$

$$\mathbf{tr}\left(Z^2W^2-(ZW)^2\right)$$

$$\mathbf{tr}\left(D_+^2Z^2-(D_+Z)^2\right)$$

$$\Delta(\lambda)$$

$$W\equiv\Phi^1+i\Phi^2,\quad Y\equiv\Phi^3+i\Phi^4,\quad Z\equiv\Phi^5+i\Phi^6$$

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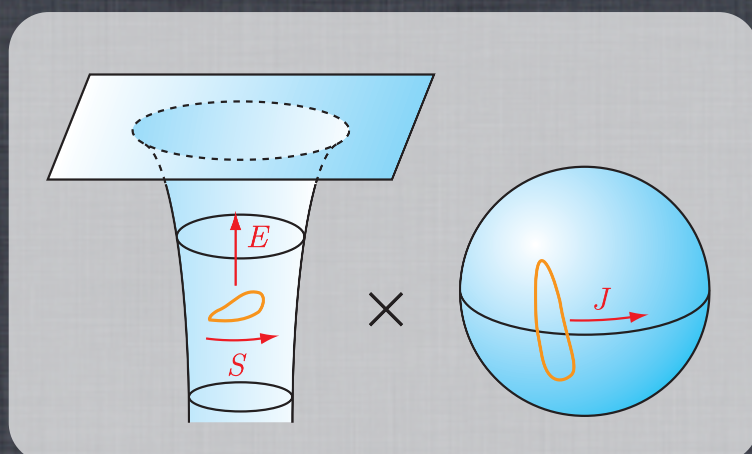

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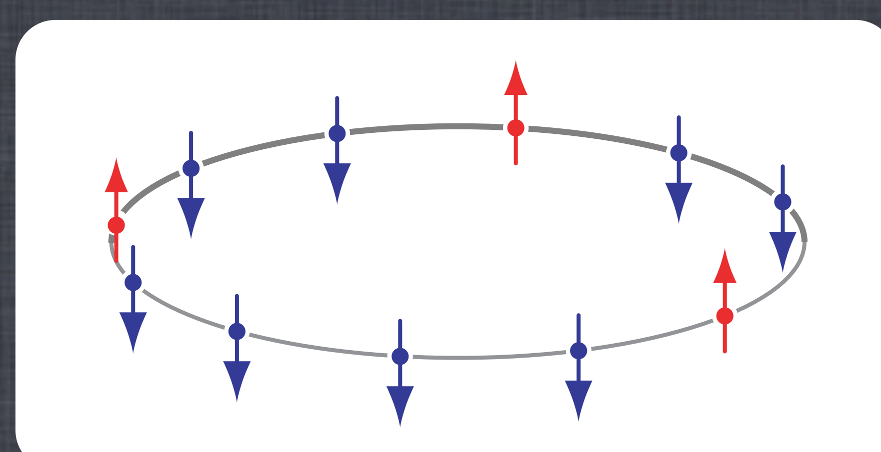
Energy of a periodic spin chain state



Exact spectrum via TBA

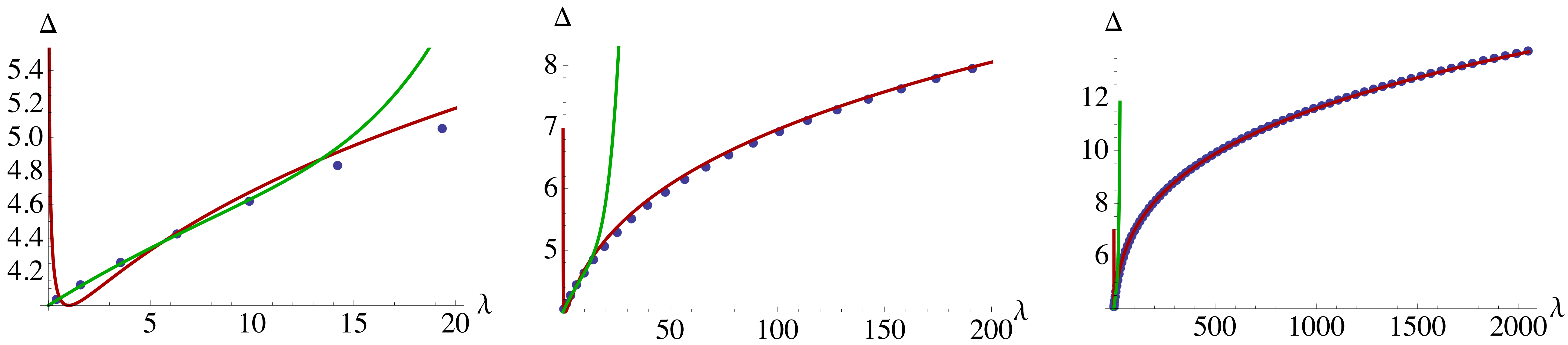
The exact Konishi dimension

- SYM results up to 5-loop

[Fiamberti, Santambrogio, Sieg, Zanon (2007)] [Velizhanin (2008)]
[Eden, Heslop, Korchemsky, Smirnov, Sokatchev (2012)]

- String results up to 1-loop

[Gromov, Serban, Shenderovich, Volin (2011)] [Roiban, Tseytlin (2011)] [Mazzucato, Vallilo (2011)]



Green: SYM, weak 5-loop   Blue: TBA, numerics   Red: String, strong 1-loop

- Numerical results up to $\lambda \lesssim 2000$

[Gromov, Kazakov, VIeira (2009)] [Frolov (2010)] and others

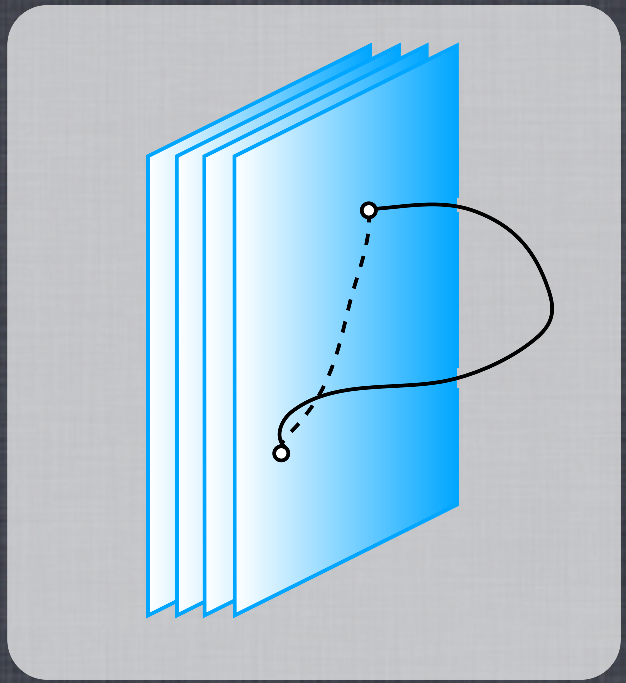
- Analytic results up to 7-loop at weak coupling

[Bajnok, Janik (2008,2012)] [Bajnok, Janik, Hegedus, Lukowski (2009)]
[Arutyunov, Frolov, RS (2010)] [Balog Hegedus (2010)] [Leurent, Serban, Volin (2012)]

Open string sector in AdS/CFT are less studied

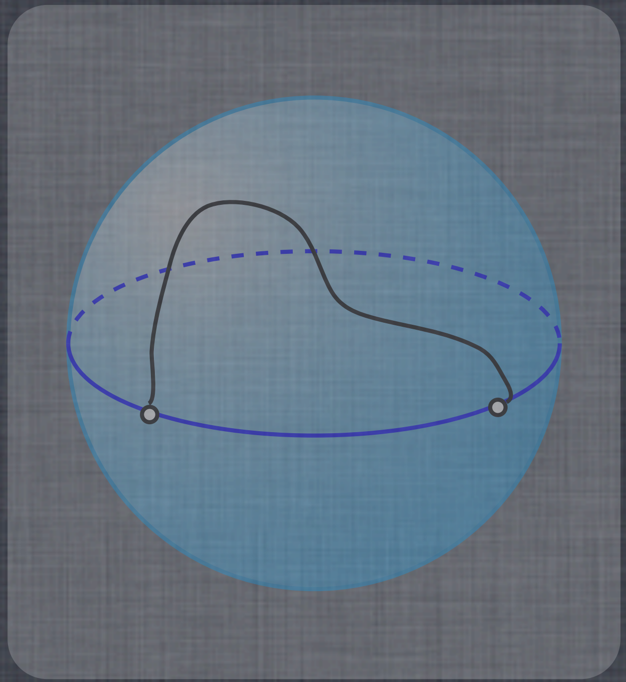
- Minimal surface vs. Wilson loop vev

An open string (or disk worldsheet) ending on a stack of N D3 branes



- Spectrum of open string state vs. Determinant-like operators

An open string ending on another rotating single D(3)-brane



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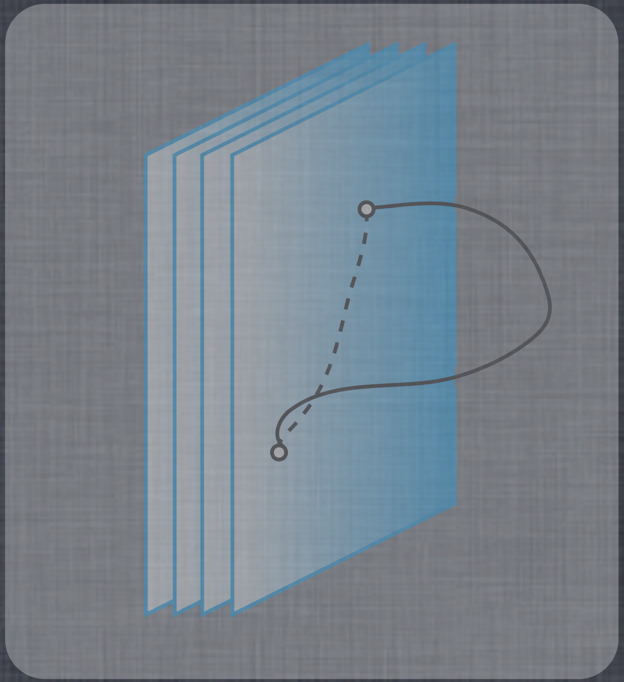

This Talk

- Spectrum of open string state vs. Determinant-like operators

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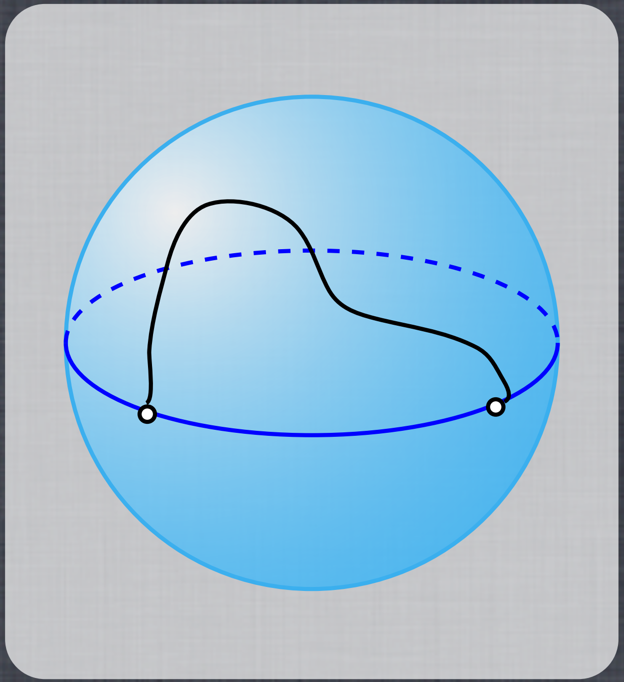

= (Spherical) Giant gravitons

- Determinant operators correspond to D-branes (without open string)

$$\det Z \equiv \epsilon_{i_1 \ldots i_N} \, \epsilon^{j_1 \cdots j_N} \, Z^{i_1}_{j_1} \ldots Z^{i_N}_{j_N}$$

$$\text{Half BPS} \quad \Rightarrow \quad \Delta_{\det} = N$$

- Determinant-like operators correspond to D-branes with open string excitations

$$\mathcal{O}_1 \equiv \epsilon_{i_1 \ldots i_N} \, \epsilon^{j_1 \cdots j_N} \, Z^{i_1}_{j_1} \ldots Z^{i_{N-1}}_{j_{N-1}} \, \chi^{i_N}_{j_N}$$

$$\mathcal{O}_2 \equiv \epsilon_{i_1 \ldots i_N} \, \epsilon^{j_1 \cdots j_N} \, Y^{i_1}_{j_1} \ldots Y^{i_{N-1}}_{j_{N-1}} \, \chi^{i_N}_{j_N}$$

$$\text{Non-BPS} \quad \Rightarrow \quad \Delta[\mathcal{O}_{1,2}] - N \;\; \text{is nontrivial}$$

Giant graviton is determinant

- Matching of the residual symmetry

$$\left[\det Z \;\leftrightarrow\; \mathrm{S}^3 \subset \mathrm{S}^5\right] : SO(6) \;\to SO(4) \times SO(2)$$

- However, multi-traces may also be good because

✓ For large operators, multi-traces can mix at large N

✓ determinant is a linear combination of multi-traces

$$\det Z = c[1^N](\mathrm{tr}\, Z)^N + \cdots + c[N]\mathrm{tr}\, Z^N, \quad c[x] = \mathrm{constant}$$

- Determinant and sub-determinant do not correlate, nor do maximal and non-maximal giant gravitons

[Witten (1998)] [Balasubramanian, Berkooz, Naqvi, Strassler (2001)] [Corley, Jevicki, Ramgoolam (2001)]

Open string in AdS/CFT from integrability

[Berenstein, Vazquez (2005)] and many others

Energy of open string ending on the D3-brane	(Subtracted) dimension of determinant-like operator
$E(\lambda)$	$\Delta(\lambda)$
	One-loop Hamiltonian is integrable

Open string in AdS/CFT from integrability

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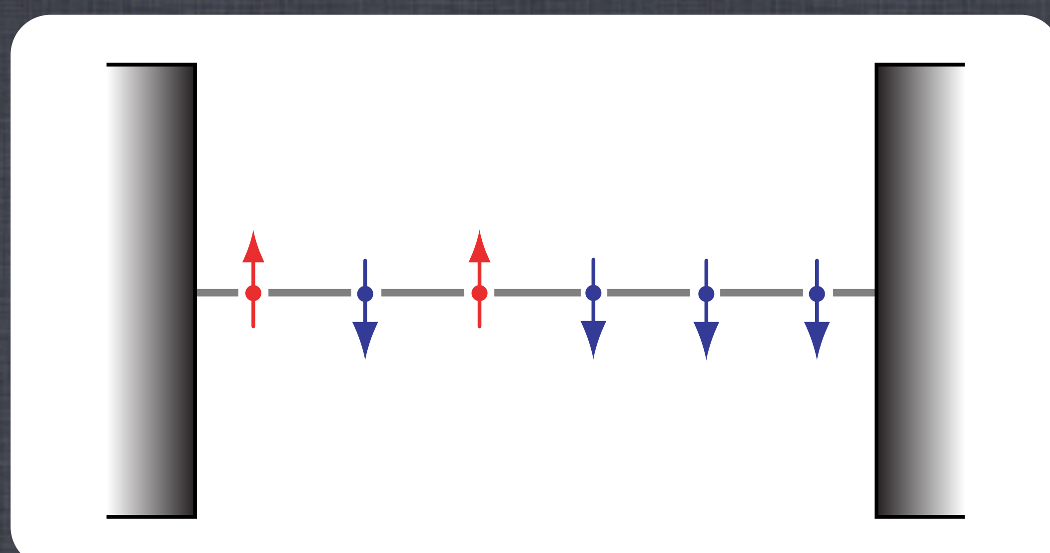
Integrability Method

Energy of an open spin chain state with integrable boundary conditions



Exact spectrum via boundary TBA?

Why boundary?

- New examples of AdS/CFT dictionary
  by applying integrability methods (TBA/Y-system ...)

- Challenge to study more general integrable models
  (periodic → twist → deformation → boundary ...)

- Boundary models are intrinsically finite-size
  (c.f. Casimir effects between parallel plates)

Our goal and strategy

Want to compute the spectrum of
an open string ending on the "Y=0" brane

[Hofman, Maldacena (2007)]

- Boundary Bethe-Yang equations
  (Asymptotic Bethe Ansatz equations)

  [Galleas (2009)]

- Finite-size corrections (Lüscher formula)

  [Correa, Young (2009)] [Bajnok, Palla (2010)]

- Conjecture the exact method (TBA/Y-system)

  [Bajnok, Nepomechie, Palla, RS (2012)]

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  [Bajnok, Nepomechie, Palla, RS (2012)]

By conjecturing how to include integrable boundaries
from the lessons in periodic (closed string) cases

Plan of Talk

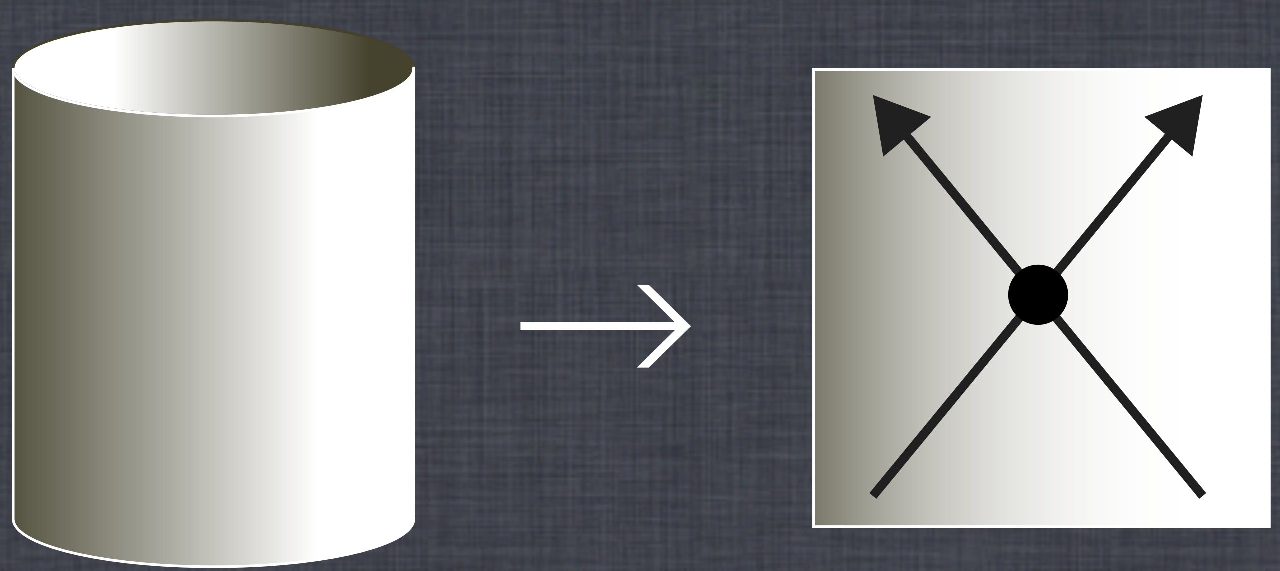
- AdS/CFT for open and closed strings
- Double-row transfer matrix
- The Y=0 brane
- Finite-size corrections from Lüscher formula
- Boundary Y-system and boundary TBA
- Conclusion

Integrable models with boundary:

double-row transfer matrix

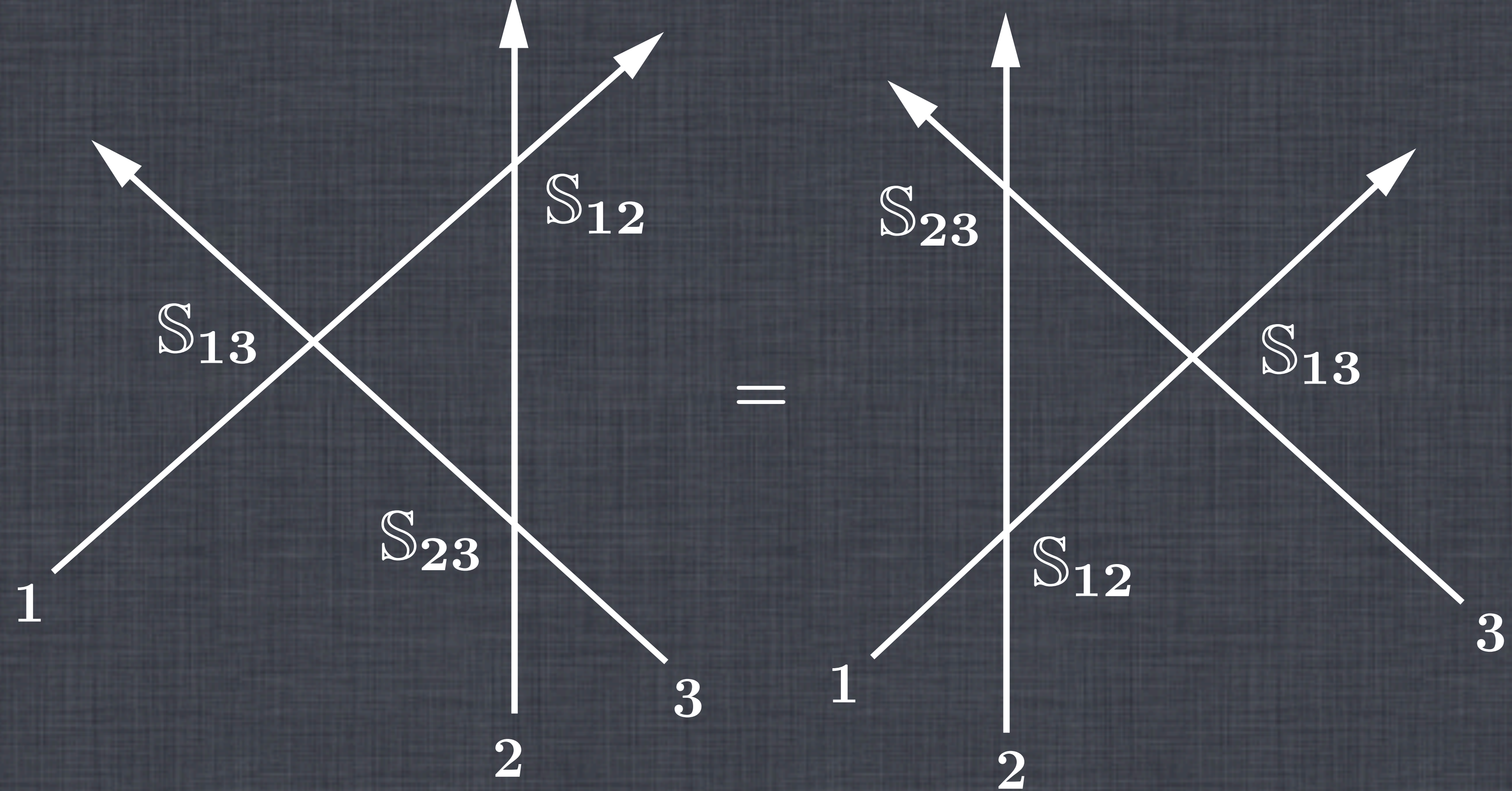
Integrability in the σ-model on $\mathrm{AdS}_5 \times \mathrm{S}^5$

- This model is classically integrable because the target space is a supercoset
- We break conformal symmetry by a gauge choice
- By taking the large-radius limit, we can define asymptotic states and their S-matrix
- This worldsheet S-matrix is (hopefully) integrable



What is integrability?

Integrable S-matrices satisfy the Yang-Baxter relation



$$\mathbb{S}_{123} = \mathbb{S}_{12}\,\mathbb{S}_{13}\,\mathbb{S}_{23} = \mathbb{S}_{23}\,\mathbb{S}_{13}\,\mathbb{S}_{12}$$

$\mathbb{S}_{ij} : \; V_i \otimes V_j \; \rightarrow \; V_j \otimes V_i \,,$ **act trivially on** $V_k \; (k \neq i, j)$

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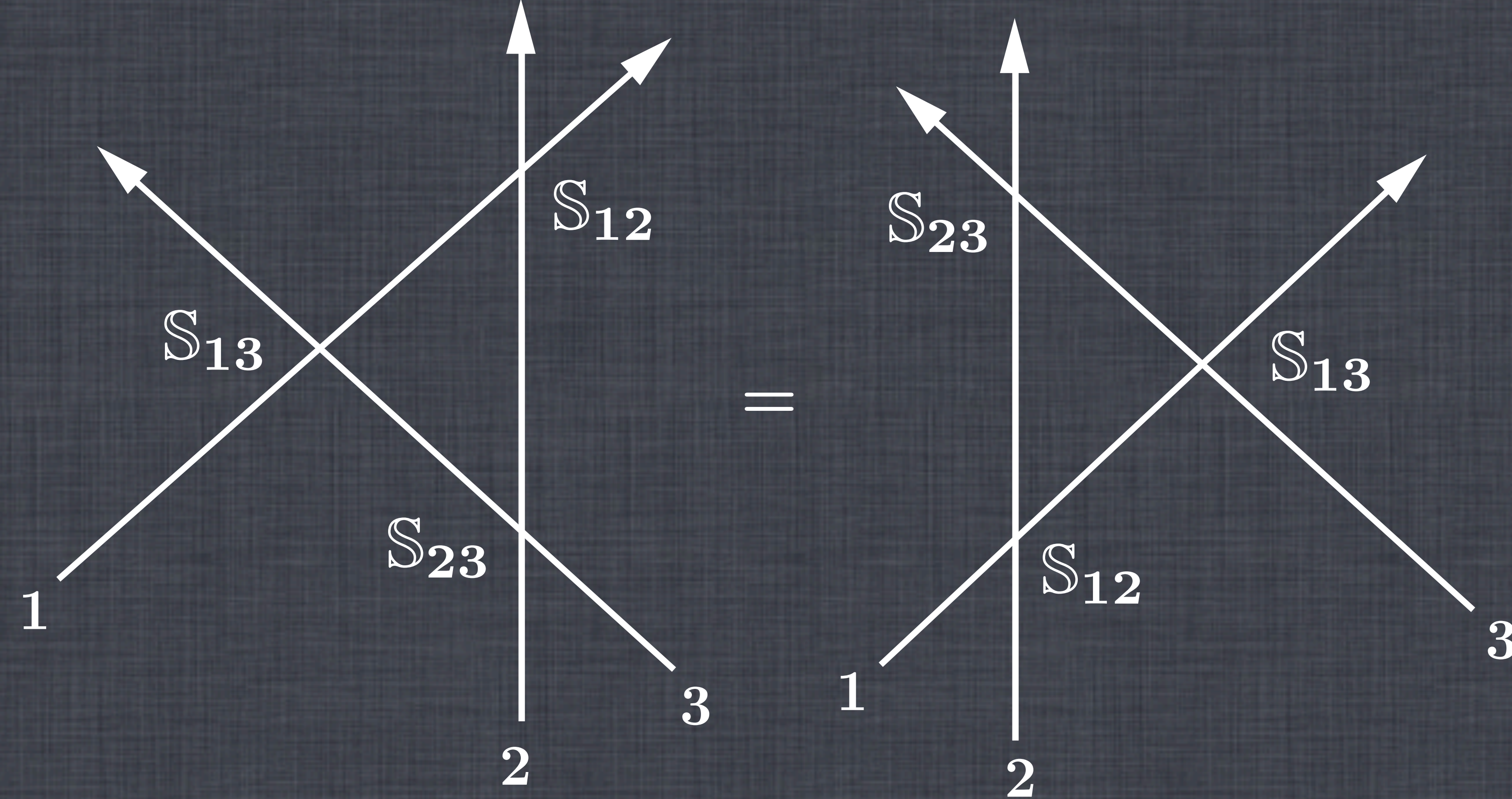
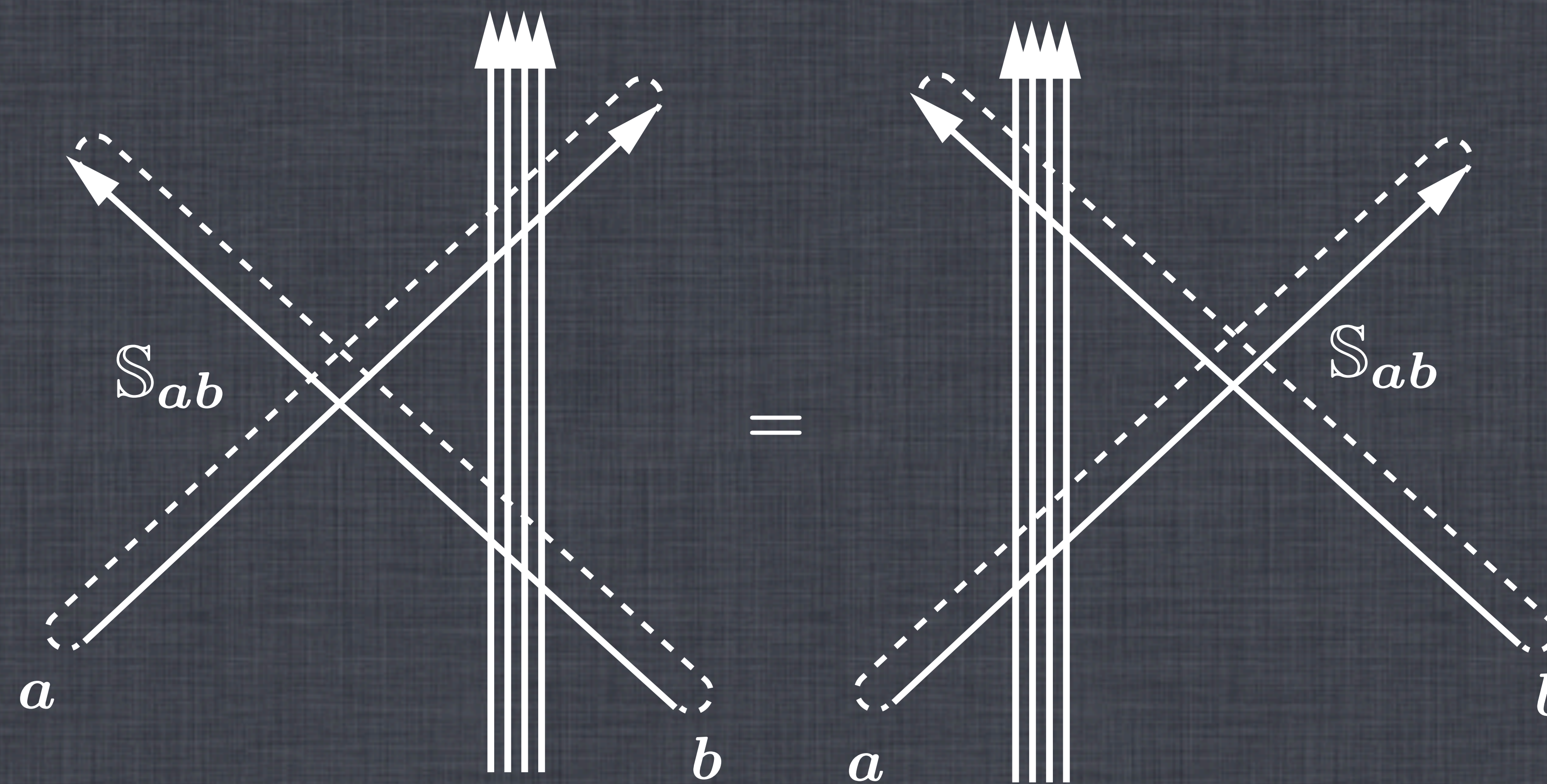


$$\mathbb{S}_{123} = \mathbb{S}_{12}\,\mathbb{S}_{13}\,\mathbb{S}_{23} = \mathbb{S}_{23}\,\mathbb{S}_{13}\,\mathbb{S}_{12}$$

Many-body S-matrix factorizes into the product of two-body S-matrices with *any* ordering of the product.

Integrability and Yang-Baxter relation

Yang-Baxter tells that transfer matrices commute



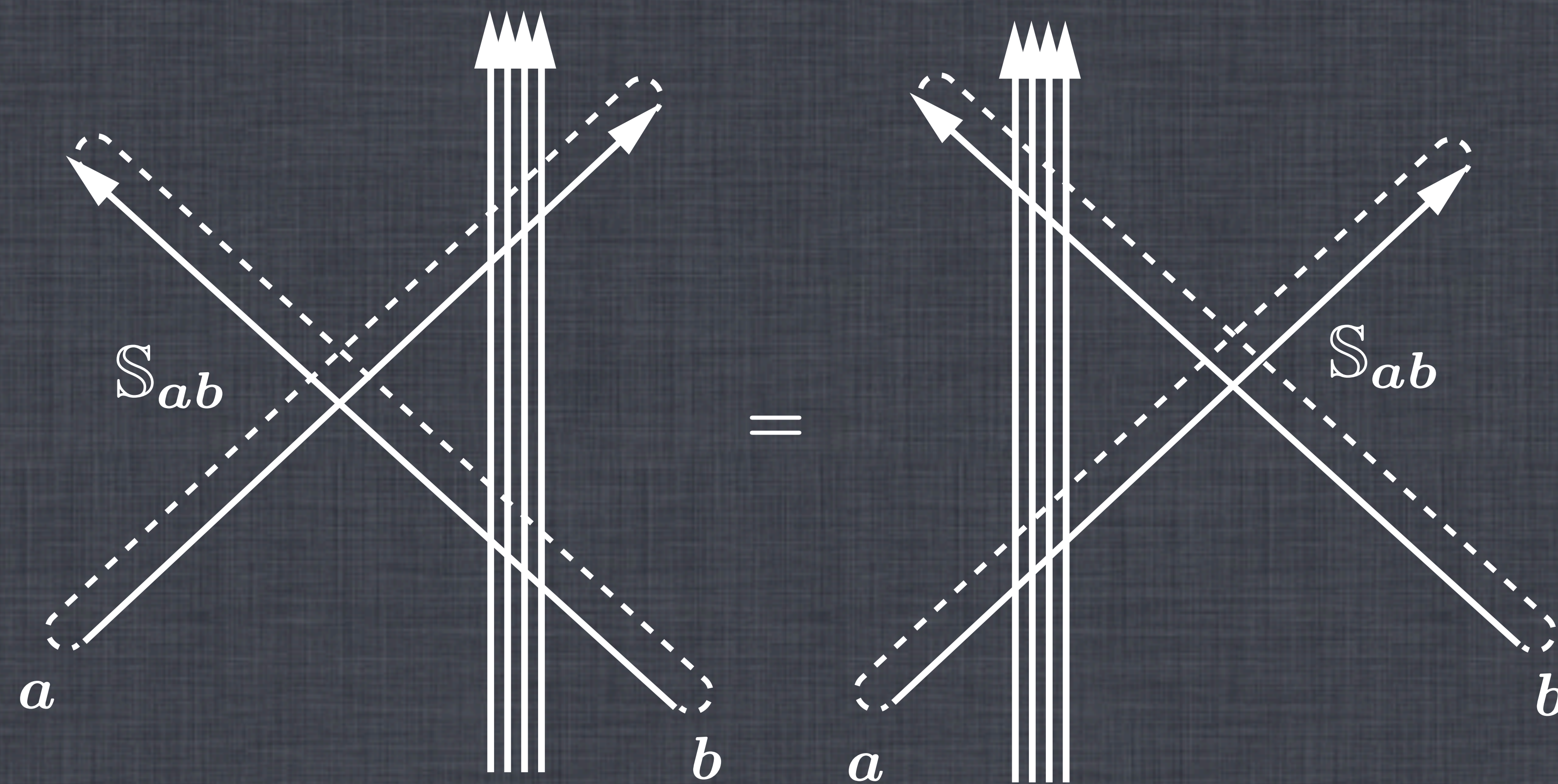
$$T_a(q) = (\mathrm{s})\mathrm{tr}_{V_a}\left[\mathbb{S}_{a1}(q,p_1)\cdots\mathbb{S}_{aN}(q,p_N)\right]$$

$$\mathbb{T}_a = \mathbb{S}_{a1}\cdots\mathbb{S}_{aN} : \; V_a \otimes V^{\otimes N} \;\rightarrow\; V^{\otimes N} \otimes V_a$$

$$T_a : \; V^{\otimes N} \;\rightarrow\; V^{\otimes N}, \quad \text{matrix of dim } V^N$$

Integrability and Yang-Baxter relation

Yang-Baxter tells that transfer matrices commute



Yang-Baxter algebra: $\mathbb{S}_{ab}\,\mathbb{T}_a\,\mathbb{T}_b = \mathbb{T}_b\,\mathbb{T}_a\,\mathbb{S}_{ab}$

Take trace in $V_a \otimes V_b \quad \Rightarrow \quad [T(q_a), T(q_b)] = 0$

$T_a(q) = \sum_n Q_n q^n$ **generates conserved charges** $\{Q_n\}$

Summary of integrability

- Yang-Baxter relation (or algebra)
- Factorized S-matrix
- Transfer matrix generates infinite charges

Transfer matrix is an important quantity in (periodic) integrable models

Summary of boundary integrability

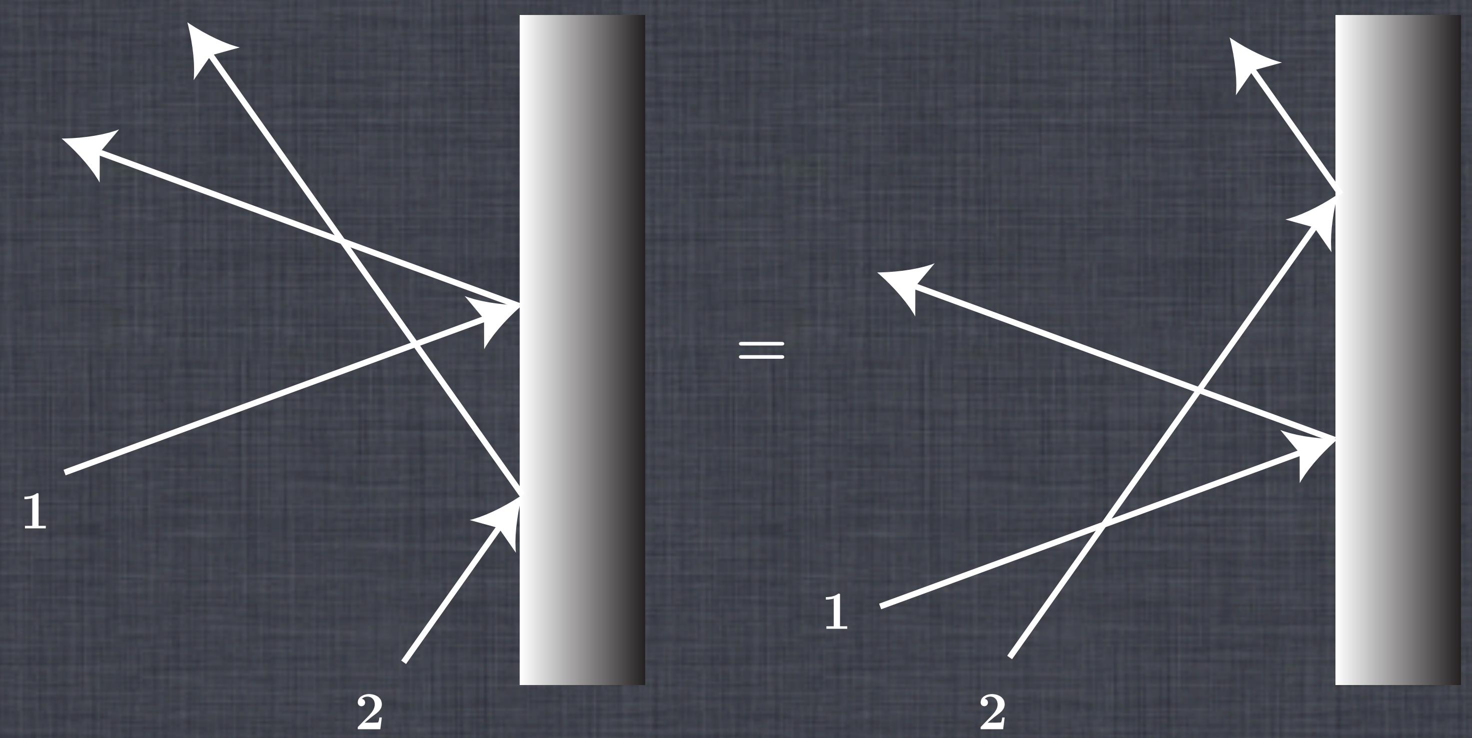
- Boundary Yang-Baxter relation (or algebra)
- Integrable reflection amplitude
- Double-row transfer matrix generates infinite charges

Double-row transfer matrix is important in boundary integrable models

Boundary Yang-Baxter relation

To maintain the integrability at boundary,
boundary reflection and bulk scattering must commute

[Sklyanin (1988)]



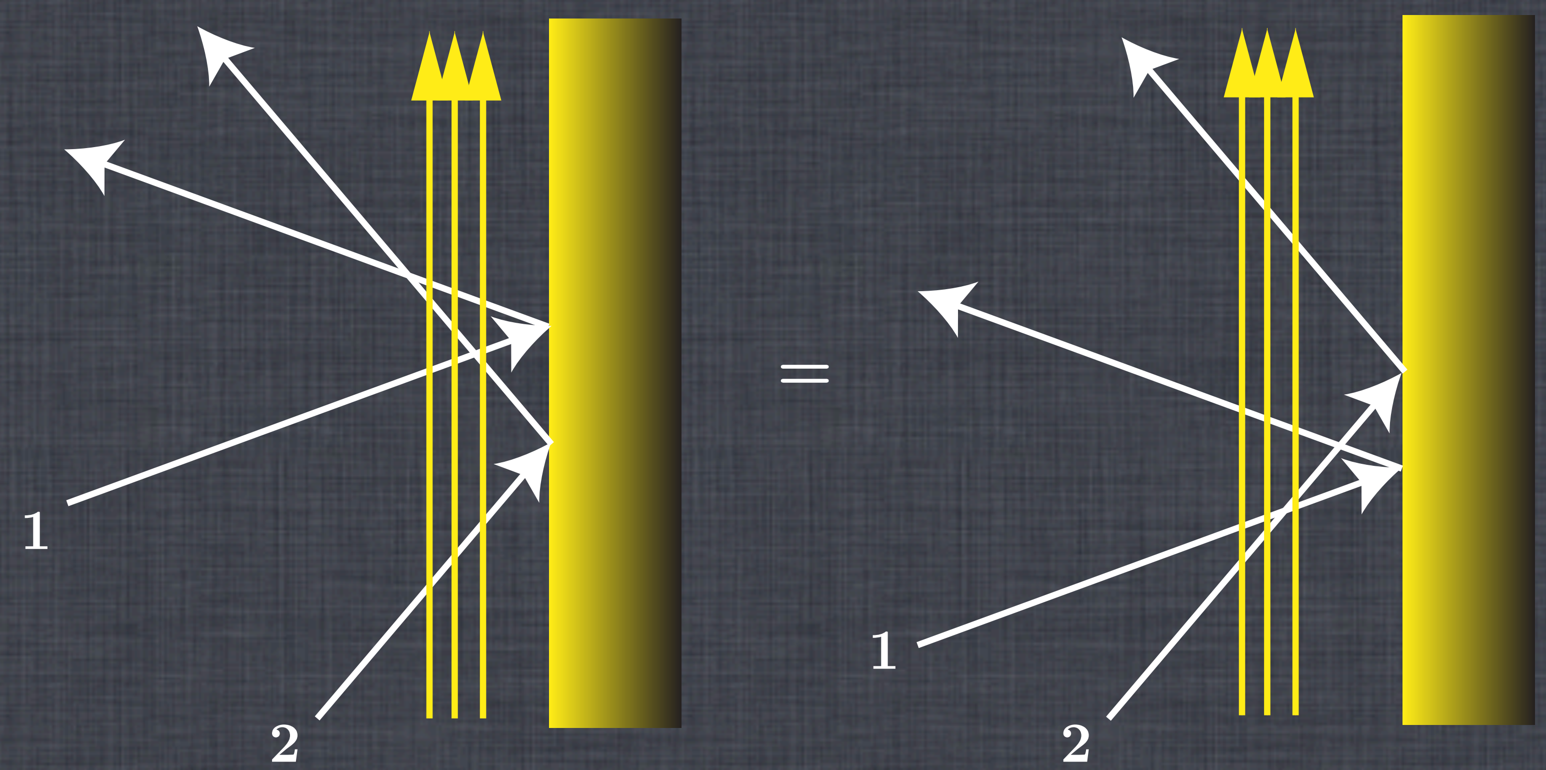
$$\mathbb{S}(-p_2,-p_1)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,-p_2)\,\mathbb{R}(p_2) = \mathbb{R}(p_2)\,\mathbb{S}(p_2,-p_1)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,p_2)$$

By using $\mathbb{S}(a,b) = \mathbb{S}(-b,-a)$ **this becomes**

$$\mathbb{S}(p_1,p_2)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,-p_2)\,\mathbb{R}(p_2) = \mathbb{R}(p_2)\,\mathbb{S}(p_1,-p_2)\,\mathbb{R}(p_1)\,\mathbb{S}(p_1,p_2)$$

Boundary Yang-Baxter relation leads to

Boundary Yang-Baxter algebra



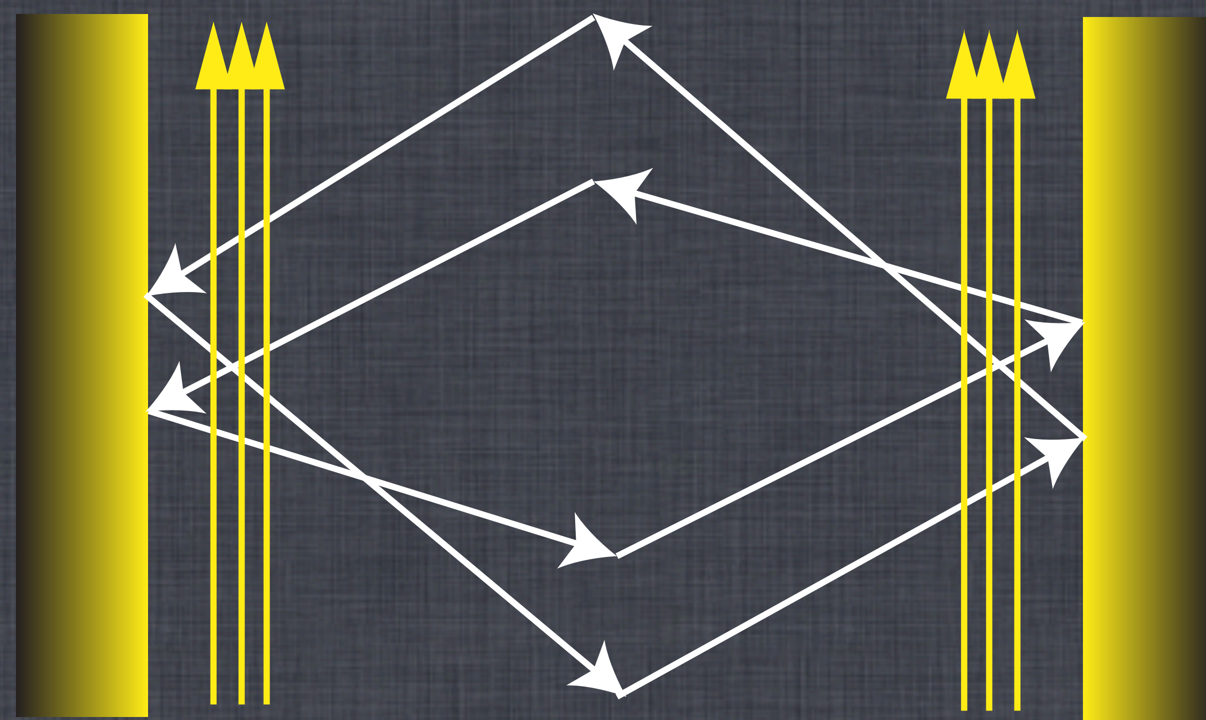
$$\mathbb{S}(p_1, p_2)\,\mathbb{T}(p_1)\,\mathbb{S}(p_1, -p_2)\,\mathbb{T}(p_2) = \mathbb{T}(p_2)\,\mathbb{S}(p_1, -p_2)\,\mathbb{T}(p_1)\,\mathbb{S}(p_1, p_2)$$

However, we cannot just take the trace !

$$\mathbb{T}(p_1)\,\mathbb{T}(p_2) \neq \mathbb{T}(p_2)\,\mathbb{T}(p_1)$$

Sklyanin combined the right- and left-reflections

[Sklyanin (1988)]



$$\mathbb{S}_{12}\,\mathbb{T}_1^{-}\,\tilde{\mathbb{S}}_{12}\,\mathbb{T}_2^{-} = \mathbb{T}_2^{-}\,\tilde{\mathbb{S}}_{12}\,\mathbb{T}_1^{-}\,\mathbb{S}_{12}$$

$$\mathbb{S}_{12}^{-1}\,\mathbb{T}_1^{+t_1}\,\tilde{\mathbb{S}}_{12}^{-1}\,\mathbb{T}_2^{+t_2} = \mathbb{T}_2^{+t_2}\,\tilde{\mathbb{S}}_{12}^{-1}\,\mathbb{T}_1^{+t_1}\,\mathbb{S}_{12}^{-1}$$

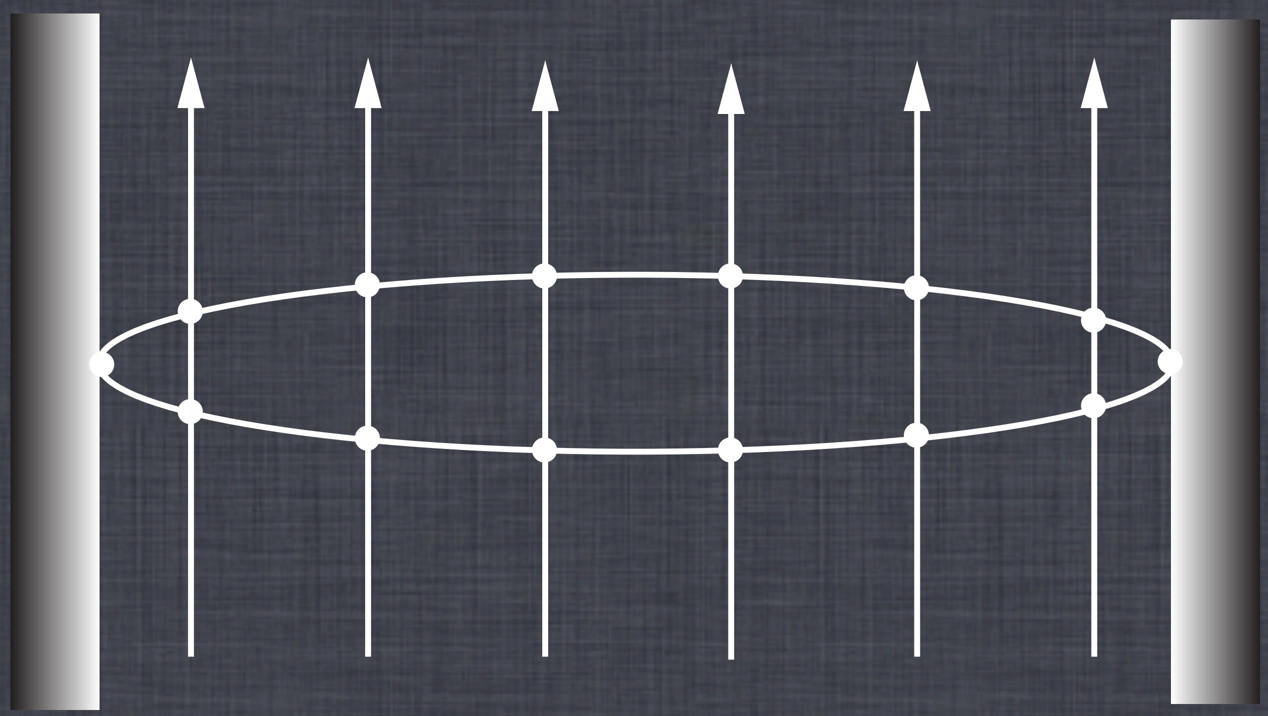
If the S-matrix is transpose invariant $\mathbb{S}_{12}^{t_1} = \mathbb{S}_{12}^{t_2}$

$D(q) \equiv \mathrm{tr}\left[\mathbb{T}_{-}(q)\mathbb{T}_{+}(q)\right]$ **with different q commute !**

Thus D generates infinite conserved charges

Double-row transfer matrix

$$D_a = \mathbf{tr}_a \Big[\mathbb{T}_- \, \mathbb{T}_+ \Big] = \mathbf{tr}_a \Big[\mathbb{S}_{aN} \cdots \mathbb{S}_{a1} \, \mathbb{R}^- \mathbb{S}_{1a} \cdots \mathbb{S}_{Na} \, \mathbb{R}^+ \Big]$$



- Da is not the "square" of transfer matrix

$$\mathbb{S}_{aj} : V_a \otimes V_j \to V_j \otimes V_a \,, \quad \mathbb{S}_{ja} : V_j \otimes V_a \to V_a \otimes V_j$$

$\mathbb{S}_{aj} \, \mathbb{S}_{ja}$ **is a matrix product**

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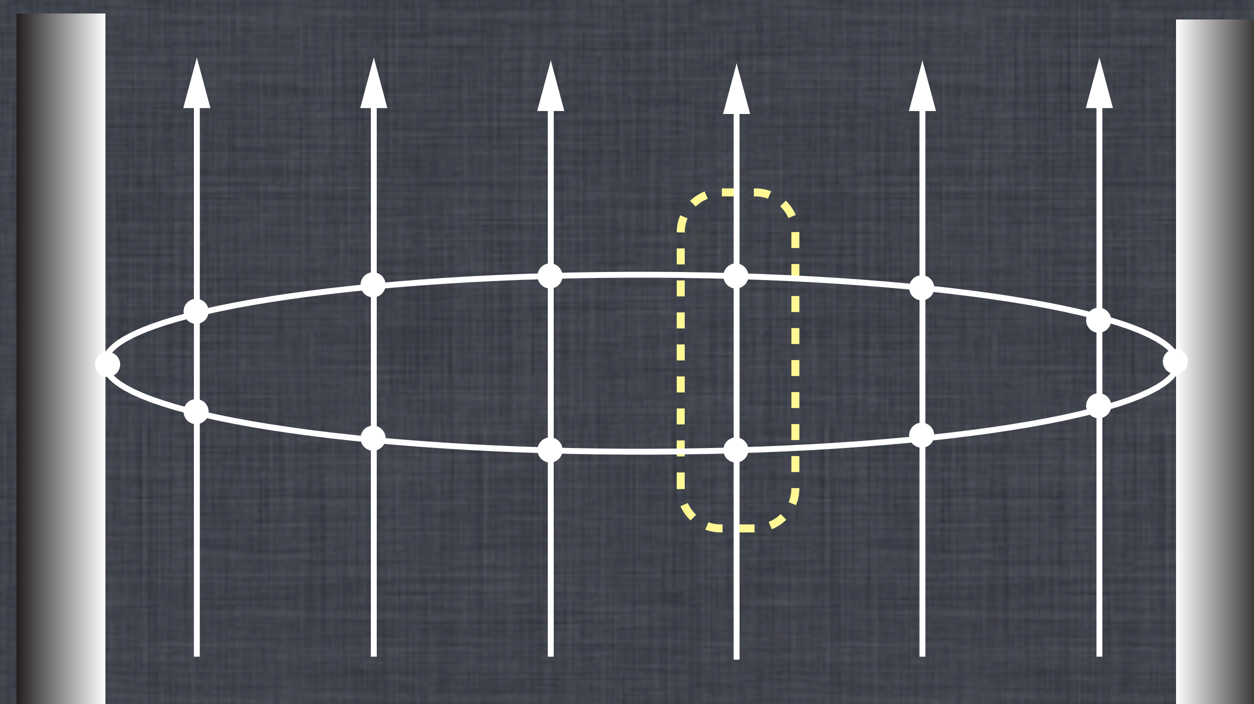

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Summary of boundary integrability

- Boundary Yang-Baxter relation (or algebra)
- Integrable reflection amplitude
- Double-row transfer matrix generates infinite charges

Double-row transfer matrix is important in boundary integrable models

The Y=0 brane

Spherical maximal giant gravitons (SMGG)

[McGreevy, Susskind, Toumbas (2000)]

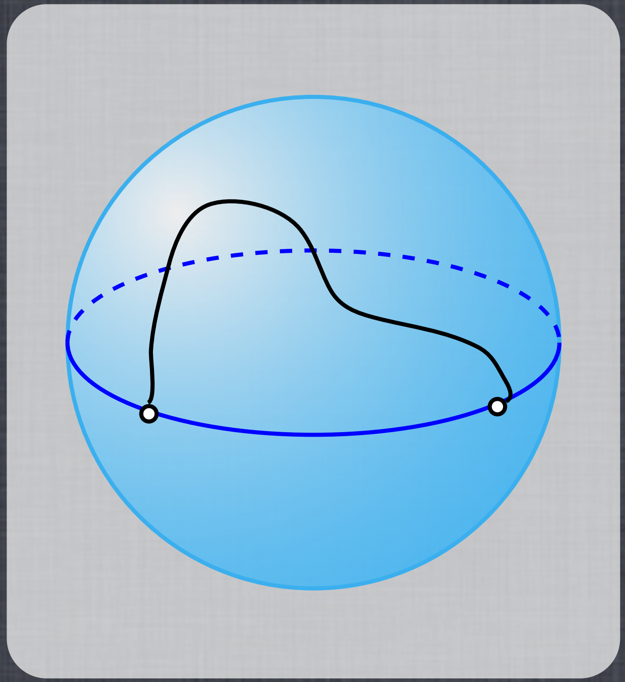
D3-brane in $AdS_5 \times S^5$

with a large angular momentum $J = \mathcal{O}(N)$

Spherical $\Leftrightarrow$ **"wrap" on $S^3 \subset S^5$**

with the angular momentum bound $J \leq N$

Maximal $\Leftrightarrow$ $J = N$ $\Leftrightarrow$ **half-BPS state**



Spherical maximal giant gravitons are dual to determinants

[Balasubramanian, Berkooz, Naqvi, Strassler (2001)]

$$\det \Phi \sim \epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} \Phi^{j_1}_{i_1} \cdots \Phi^{j_N}_{i_N}$$

Open strings on SMGG are dual to determinant-like operators

[Balasubramanian, Huang, Levi, Naqvi (2002)]

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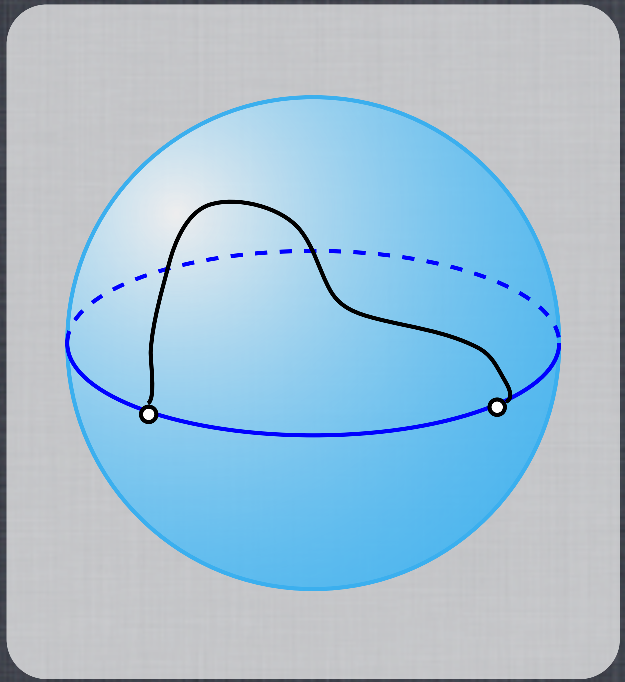

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Classification of giant graviton branes

SMGG are classified according to the choice:

$$\mathbf{S}^3 \subset \mathbf{S}^5 = \{|X|^2 + |Y|^2 + |Z|^2 = R^2\}$$

$$X = 0 \ \text{or} \ Y = 0 \ \text{or} \ Z = 0 \ \cdots$$

SMGG as a boundary condition for a spin chain

$\text{tr}\,(ZZ\cdots ZZ)$	Periodic
$\epsilon_{i_1\ldots i_N}\,\epsilon^{j_1\ldots j_N}\,Y^{N-1}\,(ZZ\cdots ZZ)^{i_N}_{j_N}$	$Y=0$
$\epsilon_{i_1\ldots i_N}\,\epsilon^{j_1\ldots j_N}\,Z^{N-1}\,(ZZ\cdots ZZ)^{i_N}_{j_N}$	$Z=0$

Insert Z^J to $\det \Phi$. The choice Z^J breaks the global symmetry

$$\mathfrak{psu}(2,2|4) \rightarrow \mathfrak{psu}(2|2)^2 \times \mathfrak{u}(1)$$

which may be broken further by boundary conditions

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$$\mathfrak{psu}(2,2|4) \ \rightarrow \ \mathfrak{psu}(2|2)^2 \ltimes \mathfrak{u}(1)$$

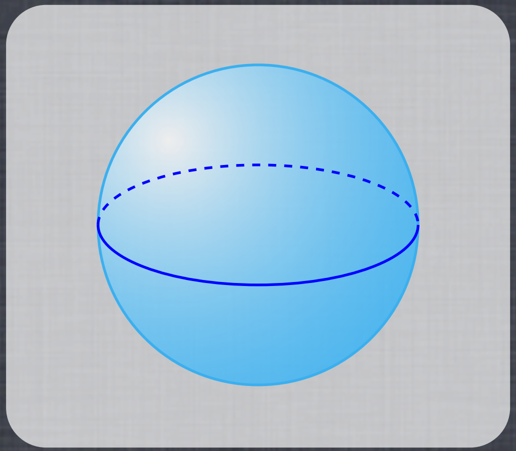
which may be broken further by boundary conditions

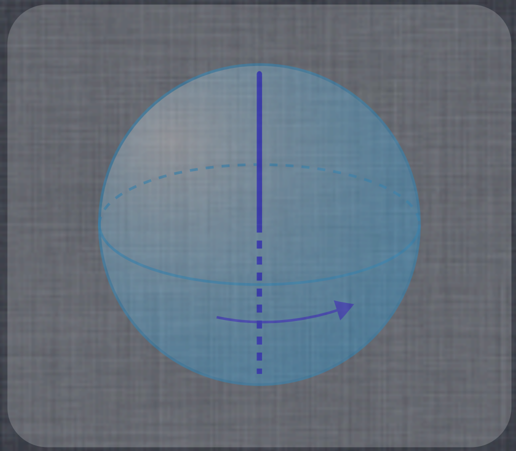
The Y=0 and Z=0 branes

[Hofman, Maldacena (2007)]

Open string state on the Y=0 brane *should* correspond to

$$\mathcal{O}_Y(\chi) \sim \sum_k \epsilon_{i_1 \ldots i_N} \epsilon^{j_1 \ldots j_N} Y^{i_1}_{j_1} \ldots Y^{i_{N-1}}_{j_{N-1}} (Z^k \chi Z^{J-k})^{i_N}_{j_N}$$





Open string state on the Z=0 brane *should* correspond to

$$\mathcal{O}_Z(\chi, \chi', \chi'') \sim \sum_k \epsilon_{i_1 \ldots i_N} \epsilon^{j_1 \ldots j_N} Z^{i_1}_{j_1} \ldots Z^{i_{N-1}}_{j_{N-1}} (\chi Z^k \chi' Z^{J-k} \chi'')^{i_N}_{j_N}$$

Unlike spinning strings, giant gravitons extends along the axis of rotation; like a electric dipole moving in the magnetic flux

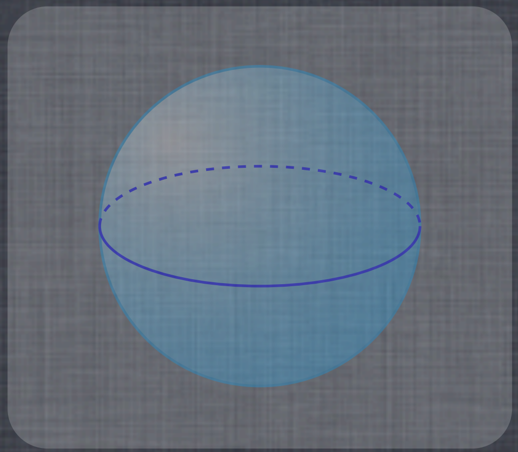
[McGreevy, Susskind, Toumbas (2000)]

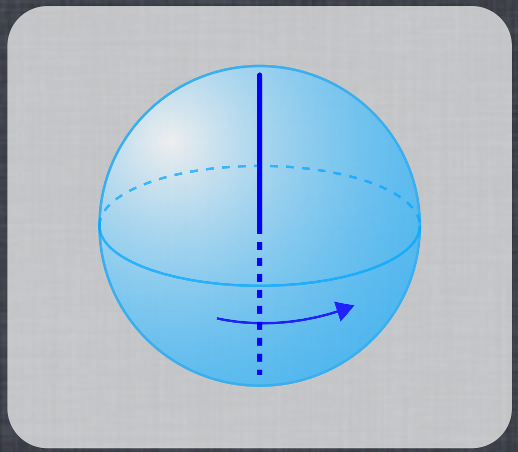
The $Y{=}0$ and $Z{=}0$ branes

[Hofman, Maldacena (2007)]

Open string state on the $Y{=}0$ brane *should* correspond to

$$\mathcal{O}_Y(\chi) \sim \sum_k \epsilon_{i_1 \ldots i_N}\, \epsilon^{j_1 \cdots j_N}\, Y^{i_1}_{j_1} \ldots Y^{i_{N-1}}_{j_{N-1}} (Z^k \chi Z^{J-k})^{i_N}_{j_N}$$





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The $Y{=}0$ branes

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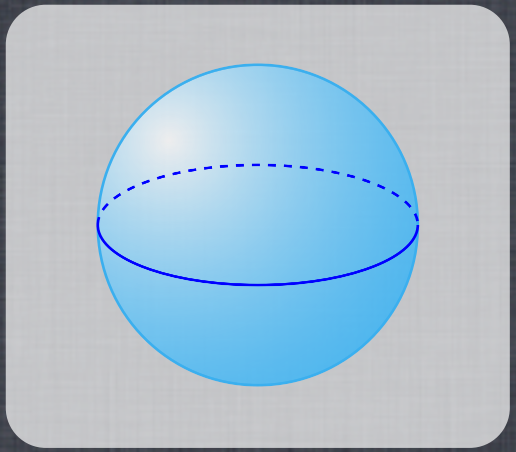
$$\mathcal{O}_Y(\chi) \sim \sum_k \epsilon_{i_1\dots i_N}\,\epsilon^{j_1\dots j_N}\, Y^{i_1}_{j_1}\dots Y^{i_{N-1}}_{j_{N-1}}(Z^k\chi Z^{J-k})^{i_N}_{j_N}$$

Preserves the symmetry $\mathfrak{psu}(1|2)^2$

No boundary degrees of freedom



$$[\mathbb{R}_Y\,, J] = 0,\ \ \forall J\in\mathfrak{psu}(1|2) \quad\Rightarrow\quad \mathbb{R}_Y \ \text{ is diagonal}$$

$$\mathbb{R}^-_Y(p) = R^-_0(p)^2 \begin{pmatrix} e^{-ip/2} & & & \\ & -e^{ip/2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\otimes 2}$$

The Z=0 branes

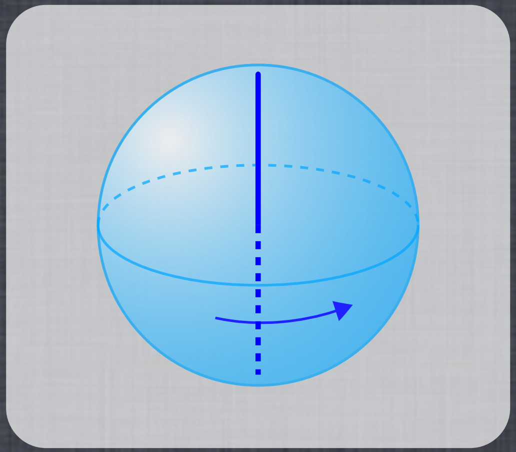
[Hofman, Maldacena (2007)]

$$\mathcal{O}_Z(\chi,\chi',\chi'') \sim \sum_k \epsilon_{i_1\ldots i_N}\,\epsilon^{j_1\ldots j_N}\; Z^{i_1}_{j_1}\ldots Z^{i_{N-1}}_{j_{N-1}}(\chi Z^k \chi' Z^{J-k}\chi'')^{i_N}_{j_N}$$

Preserves the symmetry $\mathfrak{psu}(2|2)^2$

Boundary degrees of freedom χ, χ''

(The determinant factorizes if $\chi, \chi'' = Z$ **)**



$$\mathbb{R}^-_Z : V(p)\otimes V_B \to V(-p)\otimes V_B \quad (p>0)$$

$$\mathbb{R}^+_Z : V(p)\otimes V_B \to V(-p)\otimes V_B \quad (p<0)$$

The reflection amplitude $\mathbb{R}_Z$ **is non-diagonal**

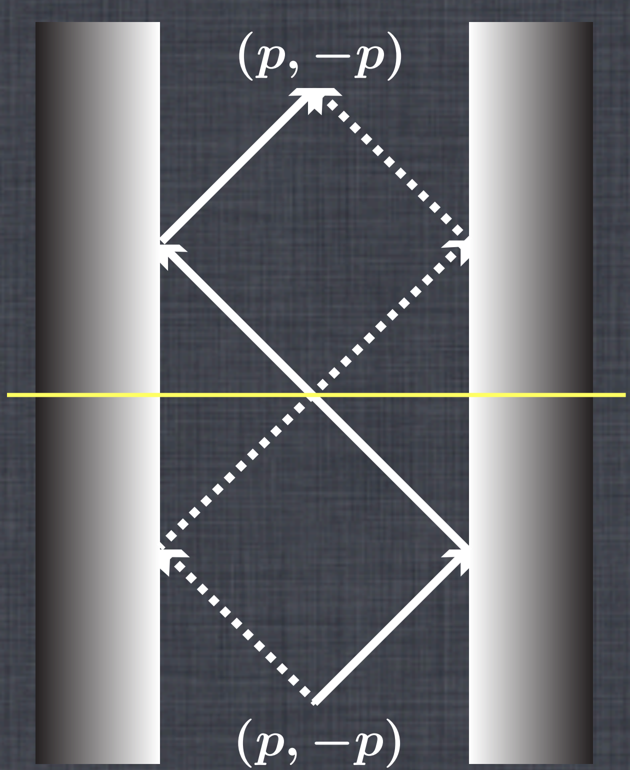
Its matrix structure can be determined by the symmetry

Boundary dressing phase

Reflection amplitude for the Y=0 brane

$$\mathbb{R}^{-}_{Y}(p) = R^{-}_{0}(p)^2 \begin{pmatrix} e^{-ip/2} & & & \\ & -e^{ip/2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\otimes 2}$$

The scalar factor is fixed by requiring that the total scattering phase of the singlet state is trivial after crossing [Beisert (2005)] [Hofman, Maldacena (2007)]



$\Rightarrow$ Boundary crossing equation

$$R^{-}_{0}(p)^2 R^{-}_{0}(-p)^2 = \frac{x^+ + \frac{1}{x^+}}{x^- + \frac{1}{x^-}} \sigma(p,-p)^2$$

A solution consistent with various limits

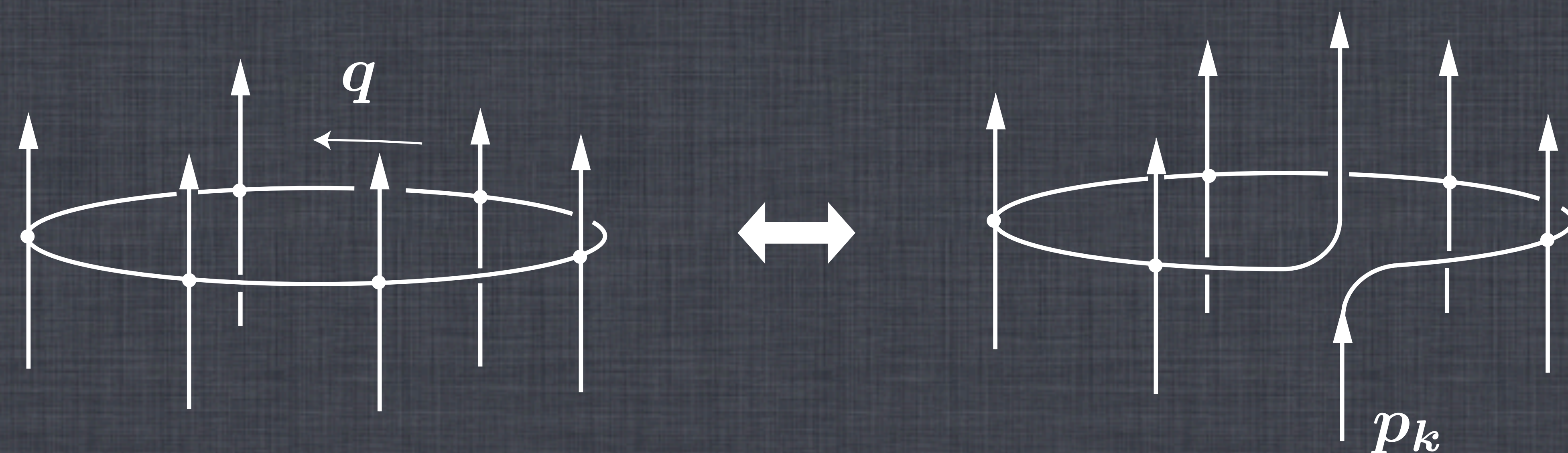
$$R^{-}_{0}(p)^2 = -e^{-ip}\sigma(p,-p)$$

[Chen, Correa (2007)]

Finite-Size corrections from Lüscher formula

Bethe Yang equations

- Transfer matrix is related to Bethe Yang equations, whose solution captures the asymptotic energy

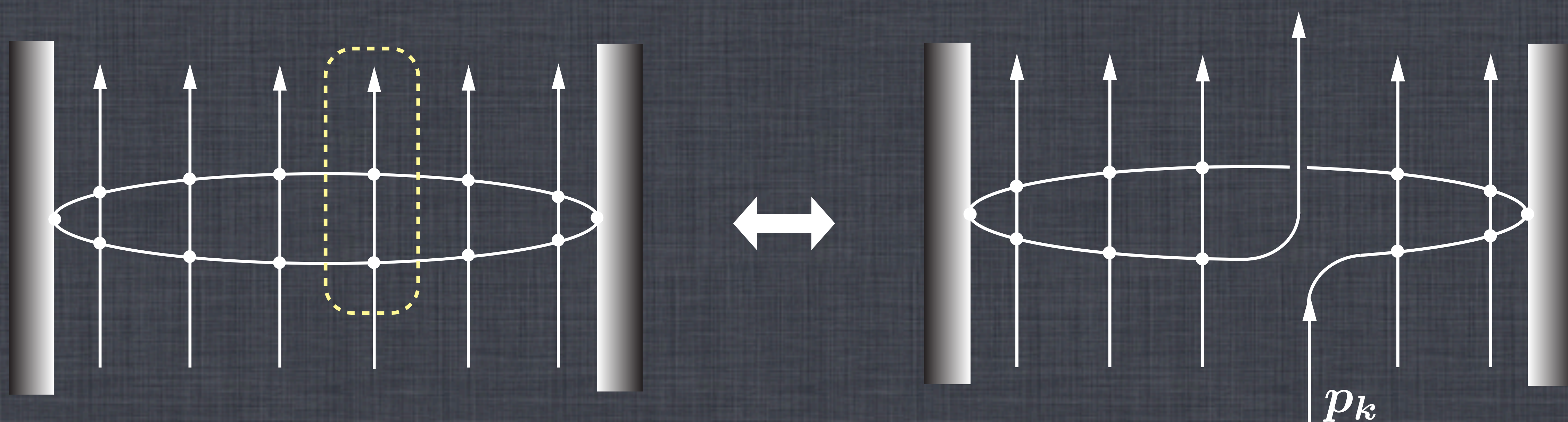


$$-1 = e^{-iJq}\, T(q|\vec{p})\Big|_{q=p_k} \quad \Leftrightarrow \quad -1 = e^{-iJp_K} \prod_{j=1}^{N} S(p_k, p_j)$$

$$E_{\text{asymptotic}} = \sum_{i=1}^{N} \sqrt{Q_i^2 + 4g^2 \sin^2 \frac{p_i}{2}}\,, \quad g = \frac{\sqrt{\lambda}}{2\pi}$$

Boundary Bethe Yang equations

- Double-row transfer matrix is related to Boundary Bethe Yang equations, whose solution captures the asymptotic energy



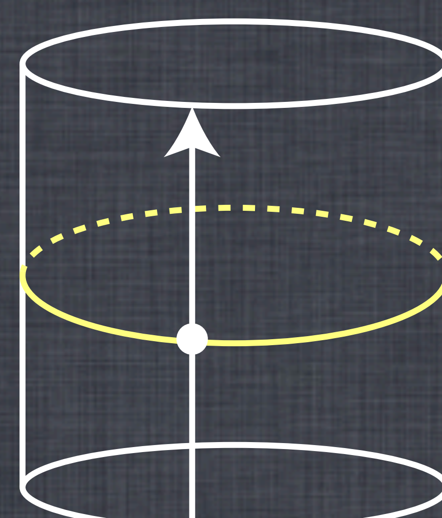
$$-1 = e^{-2iqJ} D(q|\vec{p})\Big|_{q=p_k} \quad \Leftrightarrow$$

$$-1 = e^{-i2Jp_K} \prod_{j=1}^{N} S(p_k, p_j) R^-(p_k) \prod_{j=1}^{N} S(p_j, -p_k) R^+(-p_k)$$

$$E_{\text{asymptotic}} = \sum_{i=1}^{2N} \sqrt{Q_i^2 + 4g^2 \sin^2 \frac{p_i}{2}}$$

- Bethe-Yang equations determine the asymptotic spectrum of closed string
- Boundary Bethe-Yang equations determine the asymptotic spectrum of open string
- Finite J corrections come from virtual particles in the mirror kinematics

$$\sum_Q \int d\mathcal{E}_Q \int dp_Q \, e^{-ip_Q J}$$



$$\sim \sum_Q \int d\widetilde{p}_Q \, e^{-\widetilde{\mathcal{E}}_Q(\widetilde{p}_Q) J}$$

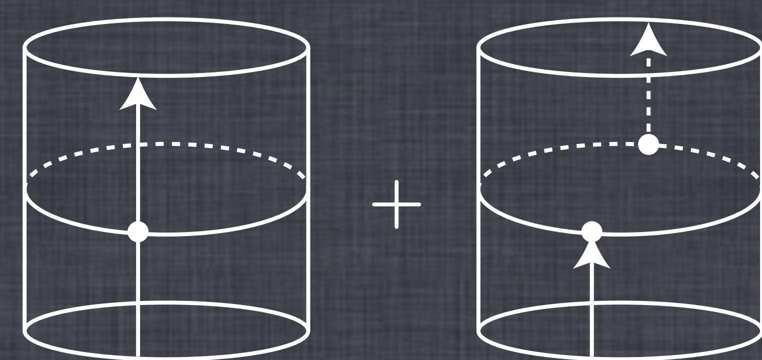
$$(\mathcal{E}_Q, p_Q) = (-i\widetilde{p}_Q, -i\widetilde{\mathcal{E}}_Q), \quad \widetilde{\mathcal{E}}_Q = 2\mathrm{arcsinh}\left(\frac{\sqrt{Q^2 + \widetilde{p}_Q^2}}{2g}\right)$$

Finite-size corrections to closed spectrum

- Lüscher formula was the main tool to study the finite-size corrections to the closed string spectrum

[Lüscher (1986)] [Janik Łukowski (2007)]

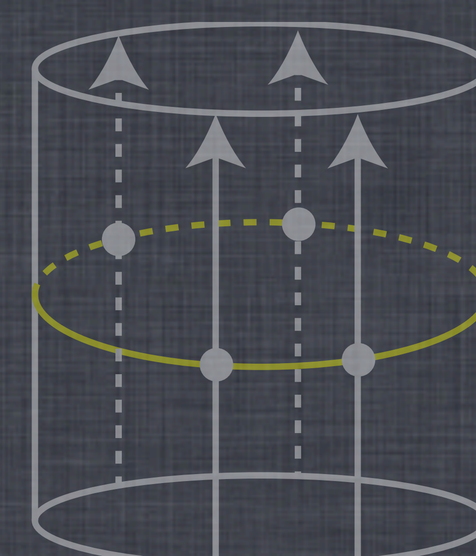
$\delta E \simeq$



- Lüscher formula is written in terms of transfer matrices

$$\delta E = -\sum_{Q=1}^{\infty}\int_{-\infty}^{\infty}\frac{d\tilde{p}_Q}{2\pi}\, Y_Q^{\circ}, \quad Y_Q^{\circ} = e^{-\tilde{\varepsilon}_Q J}\, T_Q^{2}$$

Sum over virtual particles



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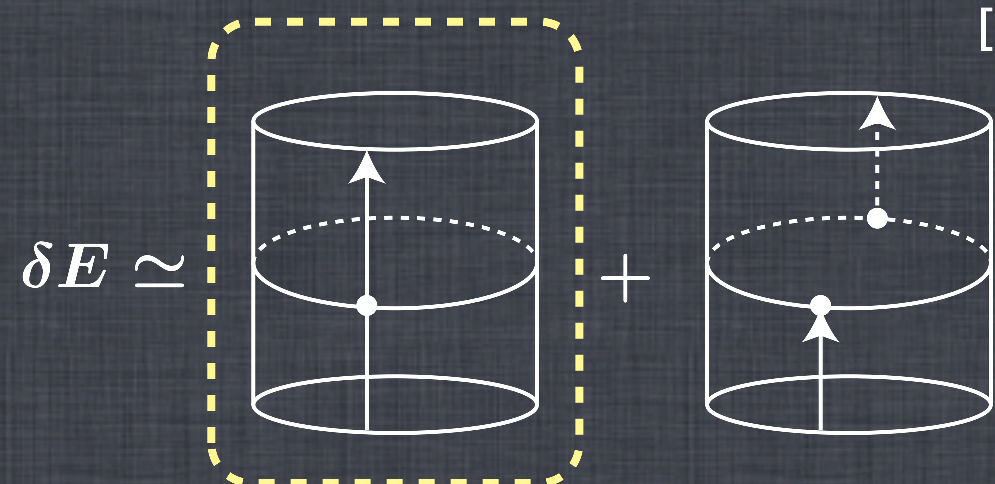

- Written in terms of transfer matrices

$$\delta E = -\sum_{Q=1}^{\infty}\int_{-\infty}^{\infty}\frac{d\tilde{p}_Q}{2\pi}\, Y_Q^{\circ}, \quad Y_Q^{\circ} = e^{-\tilde{\mathcal{E}}_Q J}\, T_Q^2$$

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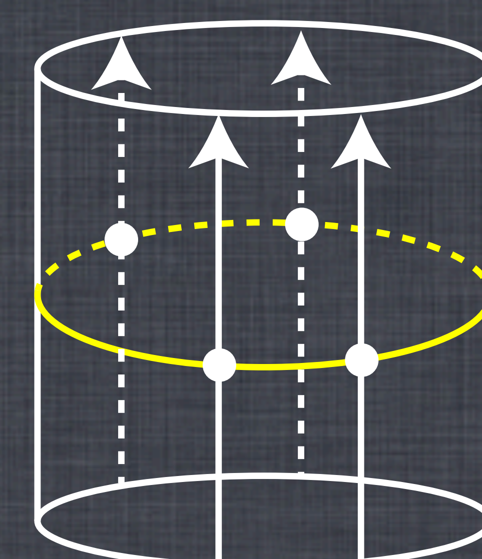
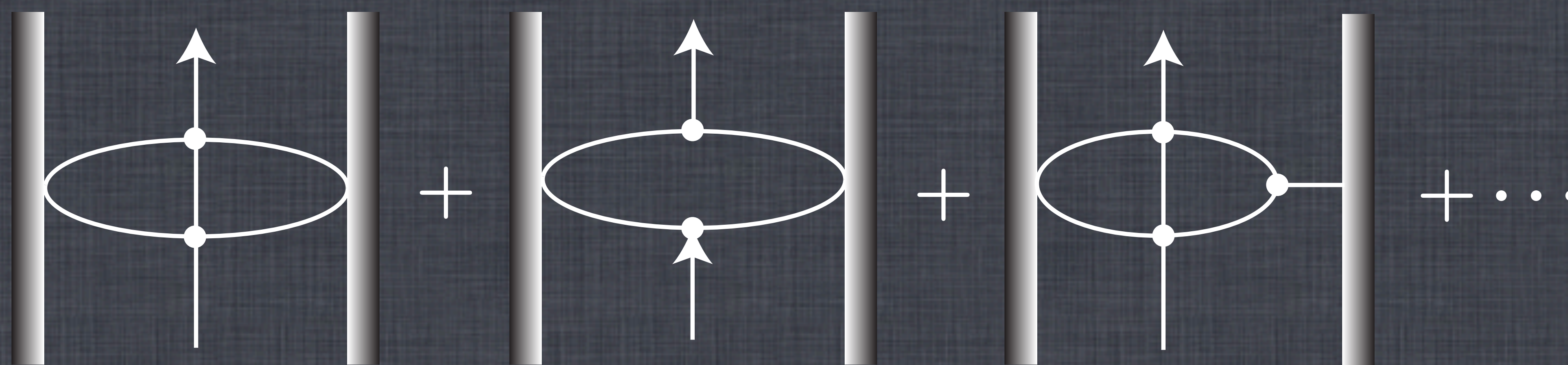


Finite-size corrections to open spectrum

- Boundary Lüscher formula has been conjectured and tested

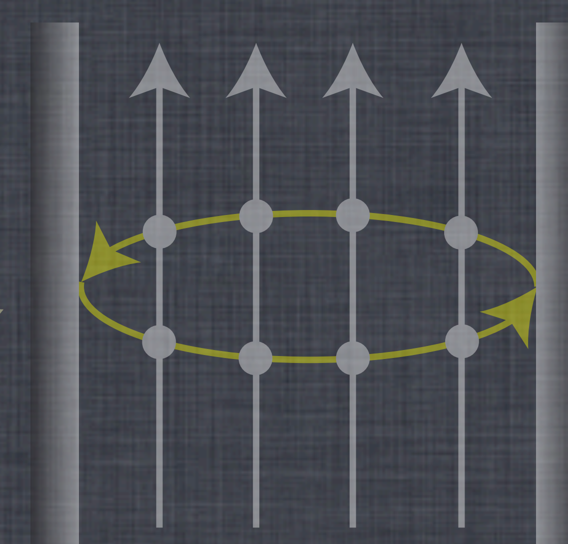
[Correa, Young (2009)] [Bajnok, Palla (2010)]

$\delta E \simeq$  $+ \cdots$

- Written in terms of double-row transfer matrices

$$\delta E = -\sum_{Q=1}^{\infty}\int_0^{\infty}\frac{d\tilde{p}_Q}{2\pi}\,Y_Q^{\circ}, \quad Y_Q^{\circ} = e^{-2\tilde{\mathcal{E}}_Q J}\,D_Q^2$$

Sum over virtual particles



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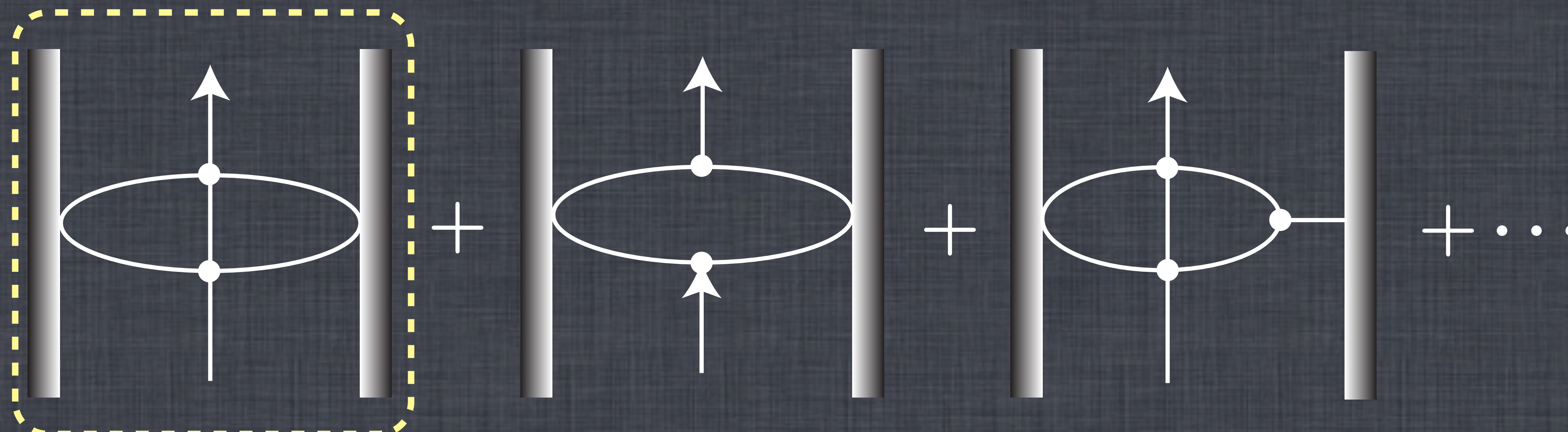

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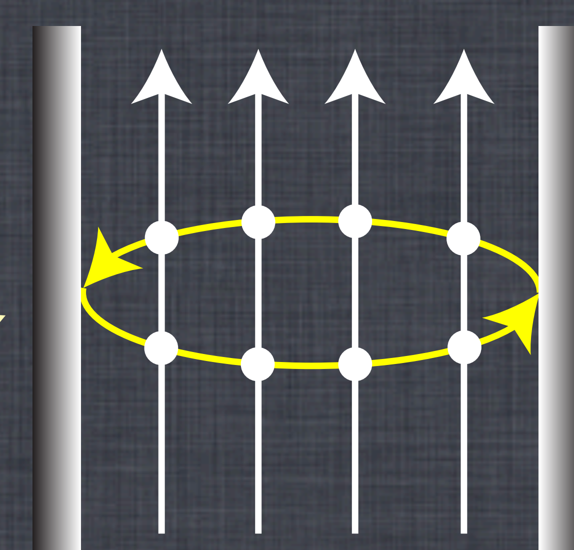


Prediction of boundary Lüscher formula

- The Y=0 ground state is BPS. Since its energy is protected, finite-size corrections vanish.

[Correa, Young (2009)]

$$\delta E[\mathcal{O}_{\mathbf{Y}}(1)] = 0$$

- The finite-size corrections to the energy of Y=0 single-particle states are nontrivial.

[Bajnok, Palla (2010)]

$$\delta E[\mathcal{O}_{\mathbf{Y}}(Y)] \approx g^{12} \cdot 192\,(4\zeta_5 - 7\zeta_9)\,, \quad \textbf{for}\ \ (J,n) = (2,1)$$

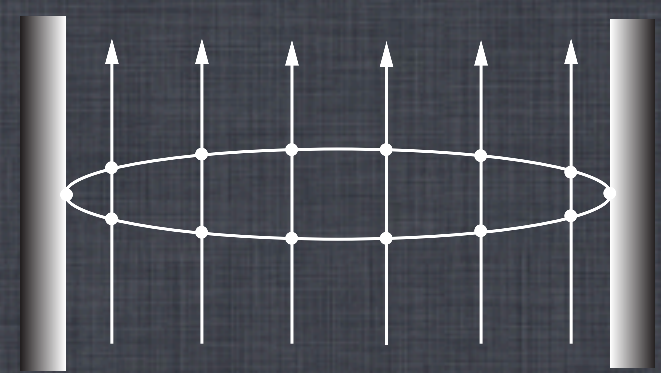
This is six-loop results in N=4 SYM. Field theoretical computation has been performed for Z=0 at four loop, but not Y=0. [Correa, Young (2009)]

$$\mathcal{O}_{\mathbf{Y}}(\chi) \sim \sum_k \epsilon_{i_1\ldots i_N}\,\epsilon^{j_1\ldots j_N}\,Y^{i_1}_{j_1}\ldots Y^{i_{N-1}}_{j_{N-1}}(Z^k\chi Z^{J-k})^{i_N}_{j_N}$$

Prediction of boundary Lüscher formula

- For general Y=0 multi-particle states, we need to diagonalize D_Q by means of algebraic Bethe Ansatz

[Arutyunov, de Leeuw, RS, Torrielli (2009)] [Galleas (2009)]



- However, the computation of the fully general case is too complicated to perform
- We conjecture the generating function for the eigenvalues of D_Q as in the periodic case

[Beisert (2006)] [Bajnok, Nepomechie, Palla, RS (2012)]

Generating function for the eigenvalues of D_Q

The su(2) sector, case of $Q=1$ [Galleas (2009)]

$$D_1 = \rho_1 \Lambda_1 + \rho_2 \Lambda_2 - \rho_3 \Lambda_3 - \rho_4 \Lambda_4$$

Bulk factor

$$\Lambda_1 = 1, \;\; \Lambda_2 = \frac{\mathcal{R}^{(-)+}}{\mathcal{R}^{(+)+}} \frac{\mathcal{B}^{(-)-}}{\mathcal{B}^{(+)-}}, \;\; \Lambda_3 = \Lambda_4 = \frac{\mathcal{R}^{(-)+}}{\mathcal{R}^{(+)+}}$$

Boundary factor

$$\rho_1 = \rho_3 = \frac{(1+(x^-)^2)(x^- + x^+)}{2x^+(1+x^+x^-)}, \quad \rho_2 = \rho_4 = \frac{x^-(x^- + x^+)(1+(x^+)^2)}{2(x^+)^2(1+x^-x^+)},$$

Notation:

$$\mathcal{R}^{(\pm)} = \prod_{i=1}^{N} \left(x(p) - x^{\mp}(p_i)\right)\left(x(p) - x^{\mp}(-p_i)\right), \quad \mathcal{B}^{(\pm)} = \prod_{i=1}^{N} \left(\frac{1}{x(p)} - x^{\mp}(p_i)\right)\left(\frac{1}{x(p)} - x^{\mp}(-p_i)\right)$$

$$x(u) + \frac{1}{x(u)} = \frac{u}{g}, \quad p_Q(u) = -i \log \frac{x^{[+Q]}}{x^{[-Q]}}, \quad f^{[n]}(u) = f\left(u + \frac{in}{2}\right)$$

$g = \frac{\sqrt{\lambda}}{2\pi}$ is coupling constant, $x = x(u)$ or $x = x(p)$

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Pairing   Pairing

$$x(u) + \frac{1}{x(u)} = \frac{u}{g}\,,\quad p_Q(u) = -i\log\frac{x^{[+Q]}}{x^{[-Q]}}\,,\quad f^{[n]}(u) = f\left(u + \frac{in}{2}\right)$$

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Generating function for the eigenvalues of D_Q

By using the eigenvalue of Q=1

$$D_1 = \rho_1\Lambda_1 + \rho_2\Lambda_2 - \rho_3\Lambda_3 - \rho_4\Lambda_4$$

the generating function for general Q is given by

$$\begin{aligned}\tilde{\mathcal{W}}^{-1} &= (1-\mathcal{D}\rho_1\Lambda_1\mathcal{D})(1-\mathcal{D}\rho_3\Lambda_3\mathcal{D})^{-1}(1-\mathcal{D}\rho_4\Lambda_4\mathcal{D})^{-1}(1-\mathcal{D}\rho_2\Lambda_2\mathcal{D})\\ &= \sum_Q (-1)^Q \mathcal{D}^Q\, D_Q\, \mathcal{D}^Q\end{aligned}$$

where $\mathcal{D} = e^{-\frac{i}{2}\partial_u} \quad \Leftrightarrow \quad \mathcal{D}f(u) = f^-(u)\mathcal{D}$

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where $\mathcal{D} = e^{-\frac{i}{2}\partial_u} \quad \Leftrightarrow \quad \mathcal{D}f(u) = f^-(u)\mathcal{D}$

$D_Q = D_{Q,1}$ corresponds to Q symmetric rep. of $\mathfrak{psu}(2|2)$

$D_{1,Q}$ for Q antisymmetric reps. of $\mathfrak{psu}(2|2)$ are generated by $\tilde{\mathcal{W}}$

We checked $D_{1,1}$, $D_{2,1}$, $D_{1,2}$ by direct computation

- Using the generating function we predicted the finite-size corrections to the energy of various Y=0 (single-particle) states, e.g.

$$\delta E[\mathcal{O}_Y(X)] \approx -2^5 \cdot g^{20}\Big[-2^3 \cdot 7 \cdot (99 - 70\sqrt{2})\zeta_9 - 2(6765 - 4785\sqrt{2})\zeta_{11} -2002(5\sqrt{2} - 7)\zeta_{15} + (7293 - 4862\sqrt{2})\zeta_{17}\Big], \quad \text{for } (J, n) = (2, 1)$$

- The result can be generalized to the full sector of $AdS_5 x S^5$

[Bajnok, Nepomechie, Palla, RS (2012)]

Boundary Y-system and boundary TBA

Generating function and T-system

$$\tilde{\mathcal{W}}^{-1} = \sum_a (-1)^a \mathcal{D}^a D_{a,1} \mathcal{D}^a, \quad \tilde{\mathcal{W}} = \sum_s \mathcal{D}^s D_{1,s} \mathcal{D}^s$$

- The generated transfer matrices solve the su(2|2)^2 T-system

$$D^+_{a,s}\, D^-_{a,s} = D_{a-1,s}\, D_{a+1,s} + D_{a,s-1}\, D_{a,s+1}$$

- We conjecture that they provide the asymptotic solutions of boundary TBA equations which gives the exact spectrum of Y=0 states

[Bajnok, Nepomechie, Palla, RS (2012)]

T-system and Y-system

The double-row transfer matrices satisfy asymptotic T-system

$$T^{+}_{a,s}T^{-}_{a,s} = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}$$

Introduce Y-functions $\quad Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}$

Y-system $\quad \frac{Y^{+}_{a,s}Y^{-}_{a,s}}{Y_{a-1,s}Y_{a+1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a-1,s})(1+Y_{a+1,s})}$

The same structure as in the closed string case !

cf. [Behrend, Pearce, O'Brien (1995)] [Otto Chui, Mercat, Pearce (2001)]

Exact energy (for open strings)

$$E_Q = \sum_{i=1}^{N}\Big(\mathcal{E}_{Q_i}(p_i) + \mathcal{E}_{Q_i}(-p_i)\Big) - \sum_{Q=1}^{\infty}\int_0^\infty \frac{d\tilde{p}_Q}{2\pi}\log(1+Y_{Q,0})$$

T-system and Y-system

The double-row transfer matrices satisfy asymptotic T-system

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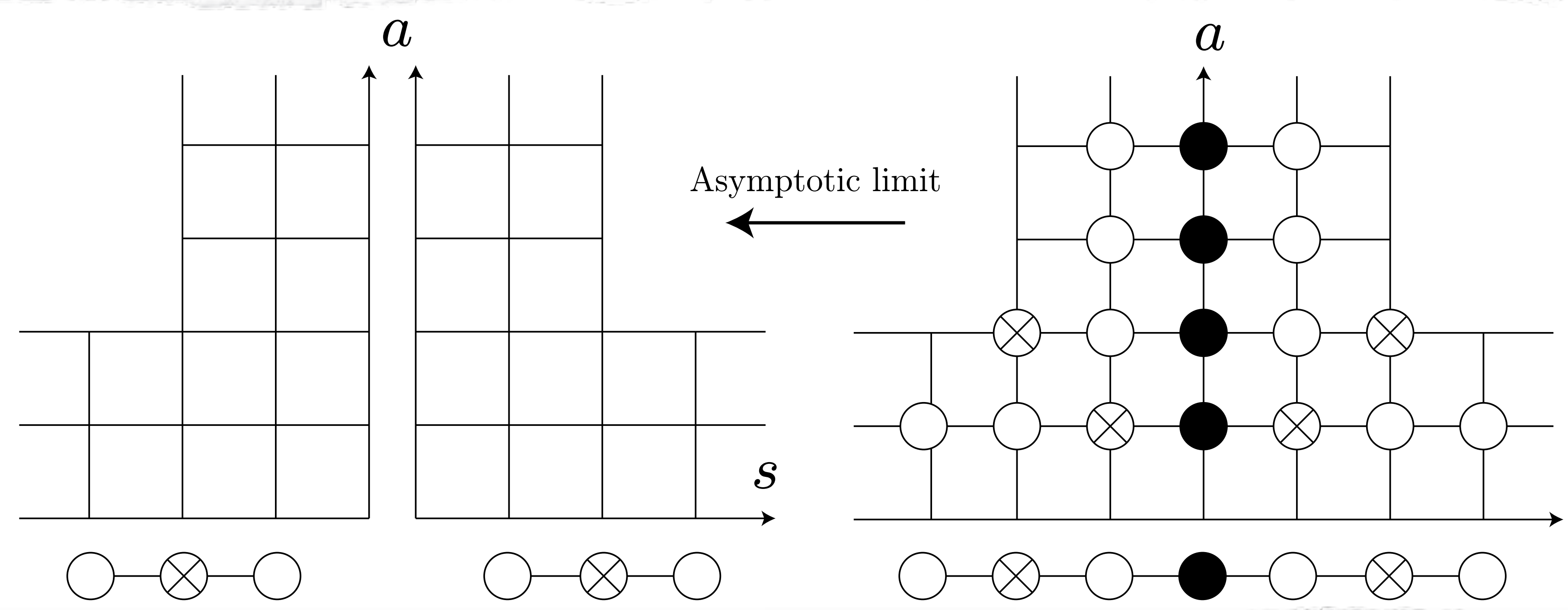
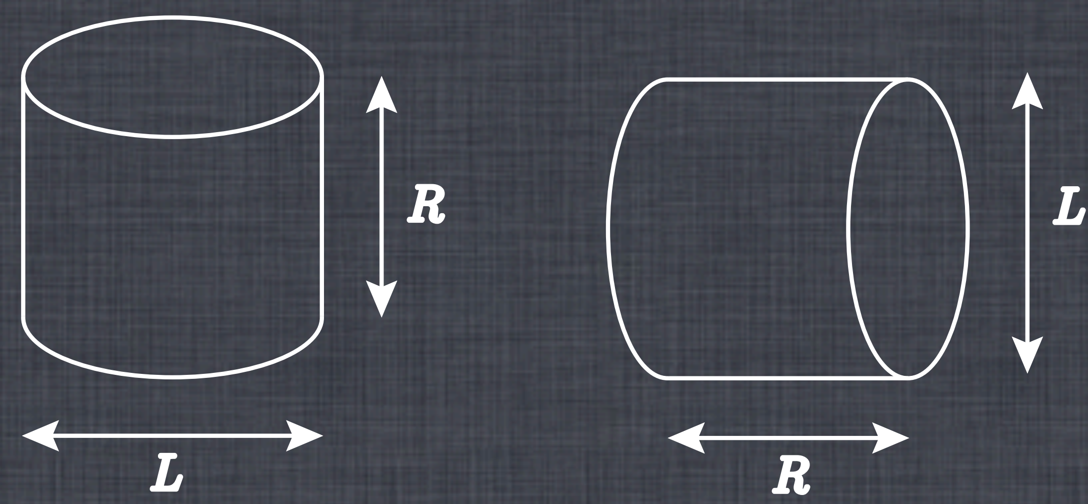


Mirror trick with boundary

- Mirror trick for periodic TBA



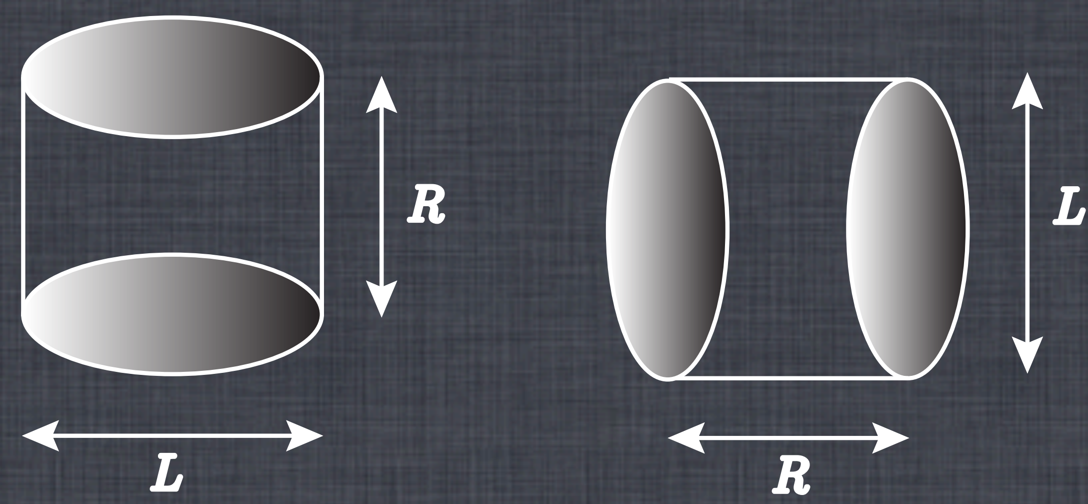
$$Z_E(L,R) = \widetilde{Z}_E(R,L) \;\to\; \exp\left(-L\mathcal{F}(\mathcal{R})\right), \quad R \to \infty$$

Extremization condition for the "mirror" free energy is called TBA equations

Typically $\log Y_a = V_a + \log(1 + Y_b) \star K_{ba}$

Mirror trick with boundary

- Mirror trick for boundary TBA



$$\langle e^{-R\,\mathcal{H}_{\ell r}}\rangle = \langle B_\ell|\, e^{-L\,\tilde{\mathcal{H}}}\,|B_r\rangle = \sum_n \langle B_\ell|n\rangle e^{-L\,\widetilde{\mathcal{E}}_n}\langle n|B_r\rangle$$

Extremize the mirror free energy with the driving term

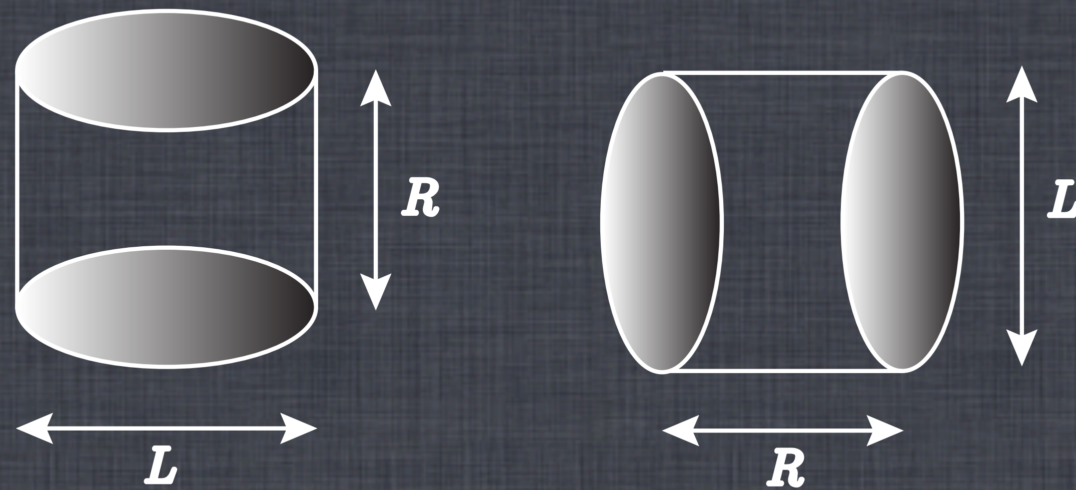
$$V_{\ell,r} = \log\left(\langle B_\ell|n\rangle\langle n|B_r\rangle\right)$$

[Leclair, Mussardo, Saleur, Skorik (1995)]

N.B. Such term often disappears when we derive Y-system from TBA

Mirror trick with boundary

- Problems to derive the boundary TBA



$$V_{\ell,r} = \log\left(\langle B_\ell | n\rangle \langle n | B_r\rangle\right)$$

However, the boundary states $|B_{\ell,r}\rangle$ are written in the Zamolodchikov-Faddeev basis instead of the Bethe Ansatz basis

These two bases are related non-trivially for the integrable models with non-diagonal S-matrix

Hence it is difficult to compute $\langle n | B_{\ell,r}\rangle$ and to derive BTBA in the AdS/CFT setup

From boundary Y-system to BTBA

- We may still conjecture BTBA for Y=0 brane
- BTBA should be same as the TBA for closed strings except for the source terms
- The source term can often be fixed by the asymptotic data
- In other words, we integrate (boundary) Y-system with (asymptotic) discontinuity relations to get/define BTBA

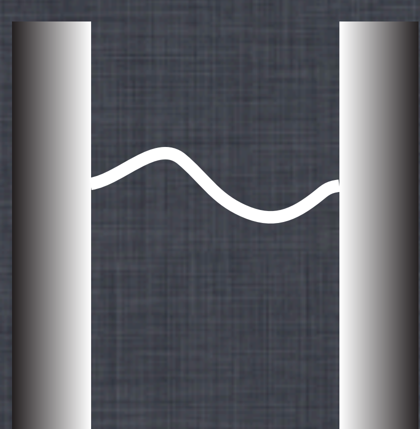
Exact energy for $Y = 0$ and $Y = 0$ & $\overline{Y} = 0$

- Since Y=0 brane is BPS, the exact ground state energy vanishes
- More interesting to study non-BPS ground states

$$\text{e.g. } Y = 0 \text{ on the left, } \bar{Y} = 0 \text{ on the right}$$

- This corresponds to changing the supertrace to the trace

- Open tachyon in the spectrum



Konishi energy $E \approx 2\lambda^{1/4} = 2\frac{R}{\sqrt{\alpha'}}$

Open tachyon energy $E \approx -\lambda^{1/4}$?

Need to solve BTBA numerically

Conclusion

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- Studied AdS/CFT for open strings ending on SMGG by using integrability methods
- Conjectured generating function for the double-row transfer matrix
- Y-system for Y=0 brane is same as Y-system for closed strings

Future directions

- Formulation of BTBA and numerical solution
- Small angle limit and analytic solution
- Rigorous derivation of integrability method
- Z=0 and other types of boundary conditions

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Thank you for attention